Dynamics of adaptive optical systems

Lukin V.P.

V.E. Zuev Institute of Atmospheric Optics SB RAS, Tomsk, Russia

lukin@iao.ru

A linear adaptive optical system is a dynamic system, and, like any dynamic system, it has a limited frequency band, i.e., a finite response time τ_a. Thus, phase measurements at time t result in changes in the phase profile of the adaptive optical element (adaptive mirror) at the later time moment $(t+\tau_a)$. Therefore, the common adaptive system is to be considered as a constant time-delay one. As a result, the problem arises of estimating the allowable time delay providing for a required level of correction, or the corresponding frequency band of the whole adaptive optical system.

Let characterize the quality of correction of optical radiation distortions in terms of Strehl ratio variation. In this case, an optical problem is formed as correction of optical wave turbulent distortions by the algorithm of phase conjugation with a reference source [1, 2]. To change from the time to spatial variables, let use the "frozenness hypothesis" for phase fluctuations, then $S(\vec{\rho}_1;\ t+\tau) = S(\vec{\rho}_1 + \vec{\upsilon}\tau;\ t)$, where $\vec{\upsilon}$ is the wind velocity vector normal to the optical wave propagation direction. Let suppose that the time delay τ is not large and, hence, use the truncate Taylor expansion. One may use the fact that the time dynamics of the process is related with the spatial one. It is also reasonable to take into account probable variations of the phase $S(\vec{\rho}_1;\ t)$ for the time τ. It is not hard to show that the correction is effective for a focused Gaussian beam with the effective diameter a, if

$$\tau \le 0.53(r_0 / v)(a / r_0)^{1/6}. \qquad (1)$$

Finally obtain, that the allowable time delay in an adaptive constant time-delay system [1, 3, 4] should be much shorter than the time of coherence radius transfer through an optical beam by a mean wind speed for a spherical wave. Note that a similar estimate of a required frequency band was made in [2] by D. Greenwood for phase conjugation adaptive systems.

For this, construct [1, 4] the correcting phase for the time point ($t+\tau$) based on the data of phase measurements at the time t for points within the transmitting aperture using the reference wave phase and its spatial derivative in the form

$$S_{corr}(\vec{\rho};\ t+\tau) = \hat{S}(\vec{\rho}+\vec{\upsilon}\tau;\ t) \approx S(\vec{\rho};\ t) + \nabla_{\vec{\rho}} S(\vec{\rho};\ t)\vec{v}\tau . \qquad (2)$$

Substituting correction term in form (2), may obtain that the allowable time delay turns out to be equal to

$$\tau_s < (r_0 / v)(a / r_0)^{7/12} \qquad (3)$$

in correction when focusing a Gaussian beam of a in radius. Comparing magnitudes of allowable times from Eqs. (1) and (3), obtain that the use of the measured phase $S(\vec{\rho},\ t)$ along with the values of its derivative [1, 4] in the correction (i.e., application of algorithm (2)) results in a significant increase in the allowable time delay providing for effective correction in comparison with the value for the common correction, namely $\tau_c / \tau \approx (a / r_0)^{5/12}$. As is seen from Eq. (3) "rapid" adaptation (18) allows significantly longer time delays as compared with the common correction [3]. The stronger phase distortions (growth of the ratio (a / r_0), the larger the gain in time in comparison with the constant time-delay correction.

Equation (3) shows that scheme (2) using both the current phase value and the current phase gradient provides for a significant increase in the allowable time delay in comparison with a common adaptive system. Note here that a wavefront sensor is usually built on the basis of the Hartmann method in real adaptive systems, and the measurement results are average phase increments on small subapertures. In view of this, there is no need to differentiate the phase measurements to obtain the phase gradient and use scheme (2). This makes the problem of using scheme (2) a correct optical problem.

To build the correcting phase for the time ($t+\tau$) from the measurements at the time t, the estimate of the phase $\hat{S}(\vec{\rho},\ t+\tau)$ on the basis of acting-time measurements of $S(\vec{\rho},\ t)$ is used in adaptive systems. As is known, the phase front evolution could be estimated the most adequately with the use of the "frozenness hypothesis":

$$\hat{S}(\vec{\rho},\ t+\tau) = S(\vec{\rho}+\vec{v}\tau,\ t)\,. \tag{4}$$

The use of such a phase estimation means that the introduced spatial distribution of the phase $S(\vec{\rho},\ t)$ is to be shifted to the value of the vector $\vec{v}\tau$ in the correction. In real conditions, the wind $\vec{v}$ randomly fluctuates; only the mean wind velocity $\vec{v}_0$ and the variance of wind component fluctuations σ_y^2 and σ_z^2 are often known in an optical experiment. Thus, obtain

$$< I(0) > = \frac{4\pi^2 a^4}{\lambda^2 L^2}\left(1 - \frac{7.04\tau^2\sigma^2}{r_0^{5/3} a^{1/3}} + \ldots\right), \tag{5}$$

i.e., limitation to the duration of time delay as

$$\tau_w \le 0.38(a / r_0)^{7/12}(r_0 / \sigma)\,, \tag{6}$$

is the condition of effective correction as well. Since $v_0 >> \sigma$ almost always, it follows from the comparison of Eqs. (1) and (6) that significantly longer time delays [1, 3, 4] are allowable in correction systems, using prediction (4), in comparison with the common correction. When comparing different schemes of adaptive optical systems operating in conditions of time constrains, we meet the problem of recalculating current phase values to the subsequent time points. In addition, this problem could be considered as the problem of predicting a random distribution of the wave phase $S(\vec{\rho},\ t)$ at the time ($t+\tau$) on the basis of optical measurements at the time t. In this case, the time delay τ is the prediction longevity for the random variable $S(\vec{\rho},\ t)$. Just such the forecasted phase $\hat{S}(\vec{\rho},\ t)$ is to serve as the correcting phase $S_{corr}(\vec{\rho},\ t)$ hereinafter.

In this work, schemes of adaptive systems have been considered, which use different time predictive algorithms of the correcting signal for future time points. It turned out that the use of predicted phase front of the correcting wave allows significantly longer time delays. The stronger phase distortions in an optical wave, the larger the gain in time in comparison with the constant time-delay correction.

References

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