Propagating aberrated laser beams

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Purpose

We outline a theory for the calculation of the laser beam quality factor of an aberrated laser beam. We provide closed form equations which show that the beam quality factor of an aberrated Gaussian beam depends on all primary aberrations except tilt, defocus and *x*-astigmatism. The model is verified experimentally by implementing aberrations as digital holograms in the laboratory. We extend this concept to defining the mean focal length of an aberrated lens, and show how this definition may be of use to people in controlling thermal aberrations in laser resonators. Finally, we look at aberration correction and control using a combination of spatial light modulators and adaptive mirrors.

Methods

The moments method has emerged as a very useful tool in the analysis of laser beams[1-3]. It is particularly useful in the calculation of parameters associated with aberrated fields. Given a Gaussian beam of size ω inside an aperture of radius *a*, whose field in normalized cylindrical coordinates is given by:

$$U(\rho,\theta) = (2\gamma^2/\pi a^2)^{1/4} \exp(-\gamma^2\rho^2)\exp(i\phi(\rho,\theta)), \tag{1}$$

where $\gamma = \omega/a$ is the truncation parameter of the aperture of radius, *a*, the phase function φ can be expanded as a linear combination of Zernike functions[4]:

$$\phi(\rho,\theta) = 2\pi\sum_{n=0}^{m} A_{n0}R_{n0}(\rho) + 2\pi\sum_{n=1}^{\infty}\sum_{n=1}^{m} R_{nm}(\rho)[A_{nm}\cos m\theta + B_{nm}\sin m\theta]. \tag{2}$$

Using the moments method, in terms of the primary Zernike coefficients, it can be shown that the beam quality factor, in the two orthogonal *x*- and *y*-axes for large γ, is given by[5]

$$M_x^4 = 1 + 24\pi^2\gamma^{-8}(B_{22}^2\gamma^4 + 3(5A_{31}^2 + 2A_{31}A_{33} + A_{33}^2 + (B_{31}^2 - B_{33}^2))\gamma^2 + 60A_{40}^2)$$

$$M_y^4 = 1 + 24\pi^2\gamma^{-8}(B_{22}^2\gamma^4 + 3(5B_{31}^2 - 2B_{31}B_{33} + B_{33}^2 + (A_{31}^2 + A_{33}^2))\gamma^2 + 60A_{40}^2). \tag{3}$$

The moments method can also be used to show that the focal length of a lens aberrated by the phase function given in Eq. 2, focusing an aberration free Gaussian beam is given by[6]:

$$f_\mp = \frac{a^2}{2\lambda}\left(-2\sqrt{3}A_{20} \mp \sqrt{6}A_{22} + 6\sqrt{5}\left(1 - 2\gamma^{-2} + 4\gamma^2(e^{2\gamma^2} - 1 - 2\gamma^2)^{-1}\right)A_{40}\right)^{-1}, \tag{4}$$

where $f_- = f_x$ and $f_+ = f_x$.

Results

The two models were tested by implementing holograms of individual aberrations using a spatial light modulator and relay imaging the beam onto a Shack-Hartmann wavefront sensor to take measurements of the respective Zernike coefficients. The results are given in the following section. The beam quality factor (Fig. 1) and the mean focal length (Fig. 2) results confirm the accuracy of the models.
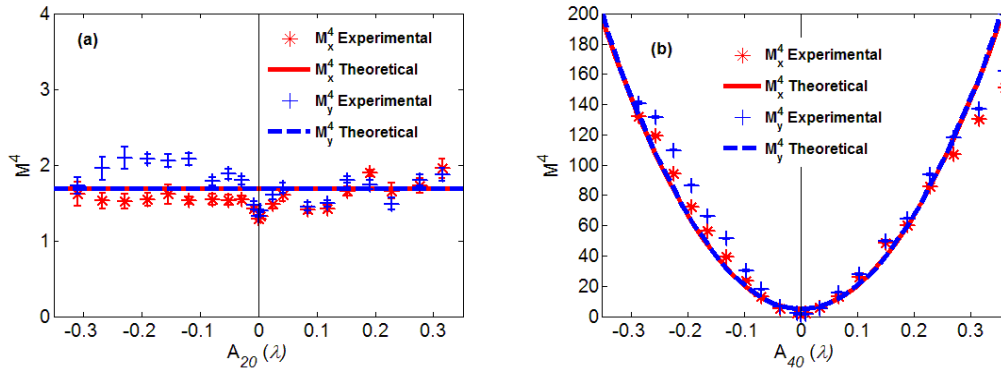
Fig. 1. The beam quality factor dependence on *rms* Zernike defocus (a) and spherical aberration (b) in the *x*-axis and *y*-axis for the model (graphs) and the experimental results (data points).
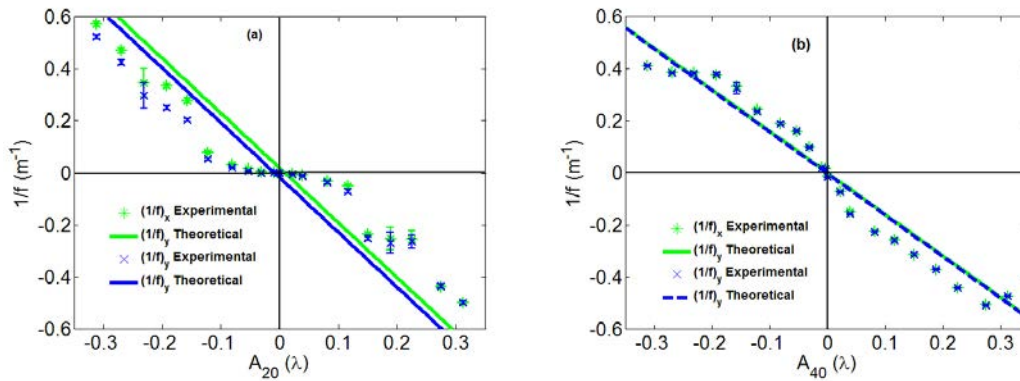

Fig. 2. Curvature dependence on defocus (a), *x*-astigmatism (b).

Conclusions

The experiments confirm the models satisfactorily. The beam quality factor model can be useful in adaptive control of optical aberrations whereas the focal length model can be very useful in the analysis of thermal aberrations of crystals in laser resonators.

References

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