

Thresholds in nonlinear Taylor Rule

C. Nebot¹, J. García-Solanes², A. Beyaert²

¹ Centro Universitario de la Defensa, UPCT cesar.nebot@tud.upct.es

² Universidad de Murcia solanes@um.es ³Universidad de Murcia arielle@um.es

Introduction

The contribution of John B. Taylor [Taylor,1993] about monetary policy rules initiated an extensive research field on instrument-based conduct of monetary policy. The original formulation of Taylor suggested that monetary authorities set short term interest rates taking into account how far current inflation is from its target and also how far current output is from its potential. In this framework, linearity assumption implies interest rate reaction to the inflation an output gaps should be symmetric and proportional

$$i_t^* = r + \pi^* + \beta(\pi_t - \pi^*) + \gamma(y_t - y^*)$$

Many researchers have been considering a variety of specifications and developing economic models for explaining the central bank decisions about the nominal interest rate in response to some macroeconomic variables. Some articles exploit non-linear estimation techniques to give evidence the Fed's target for the nominal interest rate can be non-linear.[Dolado et al., 2004] [Surico, 2007] [Petersen et al., 2007] [Cukierman, 2008] [Castro, 2011]

This paper focuses on adding spline estimations to the ongoing debate along nonlinearity and specially for detecting thresholds in monetary policy. Our strategy allows data to talk not only about parameters of a predefined model but also modelling Taylor rule based on thresholds. Multiple Adaptive Regression Splines (henceforth, MARS) is used for detecting thresholds and comparing to OLS benchmark model.

Enhancing the Taylor rule, some studies have explored how to overcome theoretical limitations. This Taylor rule considers information available so far about inflation and output gap but not necessarily future information in expected values; however, in practice, central banks tend to rely on all available information including the expected evolution of prices for defining the interest rate. Therefore, Clarida et al propose a forward-looking version of the Taylor rule where central banks considers expected inflation and output gap [Clarida et al., 1999]. Some studies have emphasized the importance of this forward-looking Taylor rule to analyze the monetary policy of the Federal Reserve and the European Central Bank [Sauer & Sturm, 2007].

Linearity of the reaction function comes out from the assumption that a central bank is minimizing a symmetric quadratic loss function regarding a linear aggregate supply function [Svensson, 1999]. However, central bank behavior may differently weight negative and positive gaps; in addition, different magnitudes of the gaps, for instance, crossing a threshold may activate a greater intensive reaction of the short term policy instrument. Central bank may have asymmetric preferences or knots in preferences and, as a consequence, follow a nonlinear Taylor rule. Indeed, Clarida and Gertler [Clarida & Gertler, 1997] rejected null hypothesis of symmetry for the Bundesbank.

Therefore, if the central bank conduct follows some thresholds preventing the magnitude of the inflation and output gaps or central bank is weighting differently negative and positive gaps in its loss function, then a nonlinear Taylor rule seems to be more adequate to explain the behavior of monetary policy. As a consequence, in the last decade, some researchers started to consider nonlinear models or asymmetries in the analysis of

monetary policy. Nonlinearity in Taylor rule due to asymmetric preferences with respect to inflation and output gaps has been explored in [Orphanides & Wieland, 2000], [Surico, 2002] [Cukierman & Muscatelli, 2008]

Some works [Assenmacher-Wesche, 2006] [Kaufmann, 2002] apply Markov-switching models to study monetary policy asymmetries and nonlinearities and they find evidences of asymmetries related to the cycles of the economy, monetary authorities tend to behave differently during recessions and expansions. Kim and Osborn [Kim et al., 2005] estimated nonlinearity in Taylor rule for the Federal Reserve but not imposing parametric assumptions and obtaining interaction between inflation and output gap. Klose [Klose, 2011] estimates an asymmetric and nonlinear Taylor rule imposing inflation target and potential output as thresholds for defining subregions. Markov and de Porres [Markov & De Porres, 2012] estimate Taylor rule for eight OCDE countries with a semiparametric procedure, generalized additive model (GAM) with and without interaction term between inflation and output gap. They find not only a strong evidence for nonlinearity in the Taylor rule for all countries but also the augmented specification with interaction term outperforms both the standard linear and nonlinear Taylor rules. Although flexibility of semiparametric procedure GAM allows to capture nonlinearity in the estimation, it does not detect possible thresholds in the Taylor rule.

The purpose of this paper consists of improving the nonlinear approach to the Taylor rule with nonparametric procedure for also detecting thresholds. For this objective, it is developed a simple model with an optimal central bank considering preferences with knots and consequently estimating thresholds in Taylor rule with Multiple Adaptive Regression Splines methodology with the interaction between inflation and output gap.

Model

According to extensive literature on the transmission mechanism of monetary policy which establishes a change in monetary policy imply a change in output in the short run and a changing inflation slowly later [Christiano, 1999] the model for the optimal behaviour of the monetary authority can be described by the minimization of the loss function subject to the aggregate supply, IS and Fisher equation..

$$\begin{aligned} & \min_{i_t} E_t \beta [\lambda g(\tilde{\pi}_{t+1}) + h(\tilde{y}_{t+1})] + E_t \beta^2 [\lambda g(\tilde{\pi}_{t+2}) + h(\tilde{y}_{t+2})] \\ \text{s. t. } & \begin{cases} \pi_{t+1} = \pi_t + \alpha \tilde{y}_t + u_{\pi,t+1} \\ \tilde{y}_{t+1} = \delta \tilde{y}_t + \eta x_t - \xi r_t + u_{y,t+1} \\ r_t = i_t - E_t(\pi_{t+1}) \end{cases} \end{aligned}$$

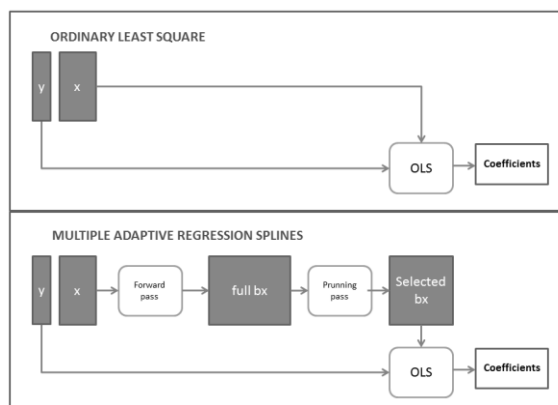
The first degree of the Taylor expansion of the Taylor Rule $i(E_t \pi_{t+1}, y_t)$ solution from the previous minimization will be:

$$i_t \simeq \left[\begin{array}{c} \alpha \lambda E_{t-1} \left(\frac{\partial^2 g}{\partial \tilde{\pi}_{t+1}^2} \right) \\ 1 + \frac{\alpha \lambda E_{t-1} \left(\frac{\partial^2 g}{\partial \tilde{\pi}_{t+1}^2} \right)}{\delta \xi E_{t-1} \left(\frac{\partial^2 h}{\partial \tilde{y}_{t+1}^2} \right)} \end{array} \right] E_t \pi_{t+1} + \left[\begin{array}{c} \alpha \lambda E_{t-1} \left(\frac{\partial^2 g}{\partial \tilde{\pi}_{t+1}^2} \right) \\ \alpha \left(1 + \frac{\alpha \lambda E_{t-1} \left(\frac{\partial^2 g}{\partial \tilde{\pi}_{t+1}^2} \right)}{\delta \xi E_{t-1} \left(\frac{\partial^2 h}{\partial \tilde{y}_{t+1}^2} \right)} \right) + \frac{\delta}{\xi} + \frac{E_{t-1} \left(\frac{\partial^2 h}{\partial \tilde{y}_t^2} \right)}{\delta \beta \xi E_{t-1} \left(\frac{\partial^2 h}{\partial \tilde{y}_{t+1}^2} \right)} \end{array} \right] \tilde{y}_t$$

Two main conclusions arise from this solution: it is important the role the second derivative of the central bank preferences plays. Indeed, thresholds will be related to changes in this second derivative of the preference function. Second, if there exist thresholds related to central bank piecewise defined preferences, $g()$ and $h()$ Taylor rule will be nonlinear containing thresholds. Therefore, any estimation of the Taylor rule should take into account either nonlinearity and the existence of possible thresholds.

Methodology

In order to estimate not only nonlinearity of the Taylor rule but also possible thresholds, we use Multiple adaptive regression spline (MARS), a nonparametric procedure that allows selecting the model and also fit this model extracting thresholds if there exist any. MARS is a methodology due to Jerome H. Friedman, for nonlinear regression modelling [Friedman, 1991] that can be conceptualized as a generalization of recursive partitioning that uses spline fitting. Given a set of predictor variables, MARS fits a model $y=f(x)+\varepsilon$ in the form of an expansion in product spline basis functions of predictors selected through a forward and backward recursive partitioning strategy. The algorithm adds a new basis $b_m(x)$ to the existing set (b^0, b_1, \dots, b_M) by conducting in a forward stepwise manner a nested exhaustive search looping over: the existing set of basis function; all predictors variables not already in the selected column; and all the values of the predictor variables not already used as knots.



So, this algorithm starts with the simplest model involving only the constant basis function; it searches the space of basis functions, for each variable and for all possible knots, and add those which maximize a certain measure of goodness of fit (minimize prediction error). It is recursively applied until a model of predetermined maximum complexity is derived. Finally, in the last stage, a pruning procedure is applied where those basis functions are removed that contribute least to the

overall (least squares) goodness of fit. Thus, this algorithm generates a sequence of combinations of basis function new variables and knots that compensates a potential new basis function. This figure illustrates a comparison between OLS and MARS procedure.

Monte Carlo simulations to observe the behaviour of the estimated coefficient demonstrate biased OLS estimators when the underlying true model is nonlinear with thresholds

Data

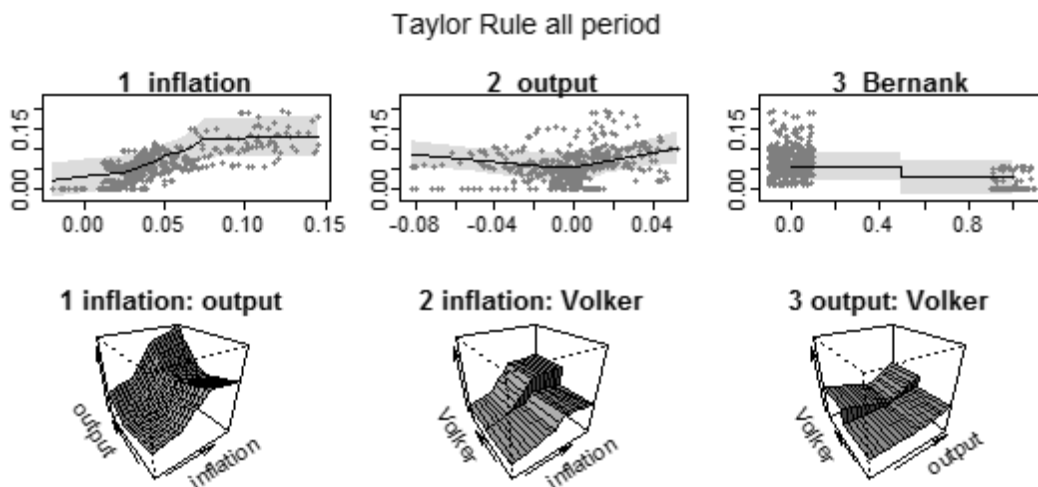
Monthly data and quarterly data from USA have been collected for the database. We have identified by dummies different periods related to different Presidents of the U.S. Federal Reserve: From 1/1970 to 7/1979 with presidents Burns and Miller is considered the "pre-Volcker period", from 7/1979 to 1/2006 as "Volcker-Greenspan period" and from 2/2006 to 1/2014 the "Bernanke period". The nominal interest rate employed is the Federal fund rate monthly and quarterly available from the Federal Reserve. The inflation rate, expressed in annual rates, is calculated from the monthly Consumer Prices Index

seasonally adjusted accessible in FRED St Louis economic data. For computing the output gap, the variables used from FRED St Louis are the quarterly real Gross Domestic Product seasonally adjusted and its potential value. For monthly values of the output gap turns to be necessary the Industrial Production Index seasonally adjusted. So, the monthly output gap is obtained from Hodrick Prescott decomposition of the logarithm of this Industrial Production Index with smoothing parameter equal to 14400.

Estimations

Here the graphic results of the contributions of each variable for the reaction function.

Monthly data



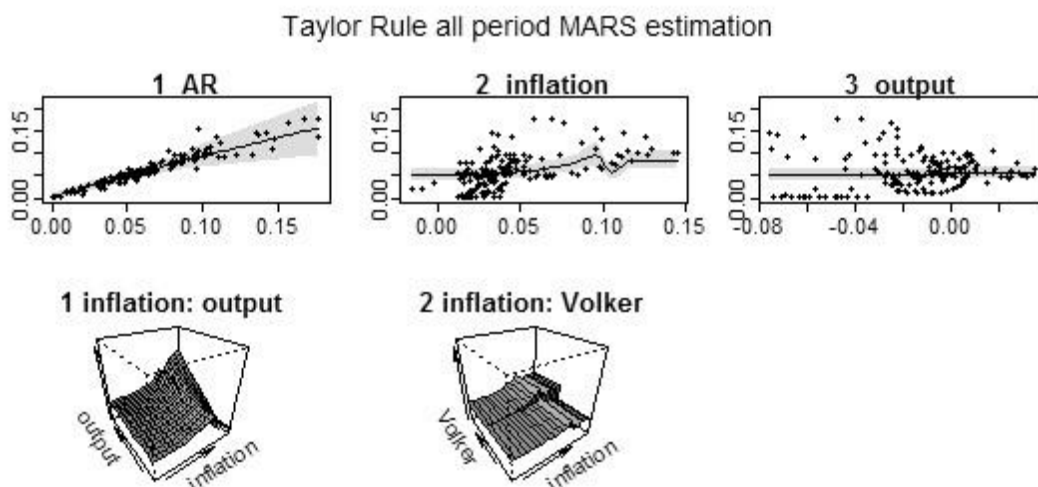
MARS adjusted R2 = 0.78

Most important thresholds: For inflation gap in 0.06195. For output gap in -0.0102

OLS adjusted R2 = 0.68

There is an increase of significance of the model with MARS and not only thresholds are detected but also an interaction between inflation and output as the model predicted.

Quarterly data a model with inertia and forward looking explanatory variables



MARS adjusted R2 = 0.9565

Most important thresholds: For inflation gap in 0.06276. For output gap in -0.05

OLS adjusted R2 = 0.9458

Conclusions

This paper presents preliminary results for the threshold estimation in the Taylor rule. A simple model describing the optimal behavior of a central bank allow us to get an approach to a nonlinear Taylor rule. Piecewise defined preference functions over inflation gap and output gap imply monetary policy with thresholds.

Multiple adaptive regression splines is employed for estimating data and it is found a significant threshold in inflation rate around 0.05 - 0.06; and also, an interaction effect between output gap and inflation rate arises as expected from the theoretical model.

From this work, the following points should be treated for a further research:

- To include the Personal Expenditure for Consumption Price Index for calculating an inflation rate target previous Bernanke period.
- To consider the Greenbook forecasts as the expected value of the explanatory variables instead of the mean of the subsequent periods.
- To include international economic variables both in theoretical model and in the econometric estimations.
- To estimate Taylor rule for ECB, Canada, UK and Australia looking for possible thresholds.

References

- Ball, L. (1999). Efficient Rules for Monetary Policy. *International Finance*, 2(1), 63–83.
- Castro, V. (2011). Can central bank monetary policy be described by a linear (augmented) Taylor rule or by a nonlinear rule? *Journal of Financial Stability*, 7(4), 228.246.
- Christiano, L. J., Eichenbaum, M., & Evans, C. L. (1999). Chapter 2 monetary policy shocks: What have we learned and to what end? volume 1, Part A of *Handbook of Macroeconomics* (pp. 65 .148). Elsevier.
- Clarida, R., Gali, J., & Gertler, M. (1999). The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature*, 37, 1661.1707.
- Cukierman, A. & Muscatelli, A. (2008). Nonlinear Taylor rules and asymmetric preferences in central banking: Evidence from the United Kingdom and the united states. *The BE Journal of macroeconomics*, 8(1).
- Dolado, J., Pedrero, R. M.-D., & Ruge-Murcia, F. J. (2004). Non-linear monetary policy rules: some new evidence for the us. *Studies in Nonlinear Dynamics & Econometrics*, 8(3).
- Dolado, J. J., Marit,a-Dolores, R., & Naveira, M. (2005). Are monetary-policy reaction functions asymmetric?: The role of nonlinearity in the phillips curve. *European Economic Review*, 49(2), 485.503.
- Friedman, J. H. (1991). Multivariate adaptive regression splines. *The Annals of Statistics*, 19(1), 1.67.
- Keogh, G. (2010). *Univariate Time Series Modelling and Forecasting using TSMARS*. Theory of probability, stochastics, mathematical statistics. LAP LAMBERT Academic Publishing.
- Svenson, L. (1997). Inflation forecast targeting. *European Economic Review*, (pp. 111–146).
- Svensson, L. E. (1999). Inflation targeting as a monetary policy rule. *Journal of monetary economics*, 43(3), 607–654.
- Taylor, J. B. (1993). Discretion versus policy rules in practice. In *Carnegie-Rochester conference series on public policy*, volume 39 (pp. 195–214).: Elsevier.