anales de psicología
2008, vol. 24, nº 2 (diciembre), 180-188

ISSN edición impresa: 0212-9728. ISSN edición web (www.um.es/analesps): 1695-2294

TEMA MONOGRÁFICO: Psicología de las matemáticas

Why children differ in their mathematical attainment at primary school?

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Abstract: Children differ considerably in their mathematical attainment in primary school. This paper reviews studies indicating the importance of socioeconomic status and heredity before considering basic calculation, the addition of single digit numbers and corresponding subtractions. Basic calculation proficiency varies with more general mathematical attainment. Delayed development of basic calculation proficiency is common in groups of children making slow progress in number whether or not they are also delayed in reading. Basic calculation might be important for mathematical development because it underpins arithmetic. Alternatively basic calculation might simply be a marker for mathematical development because the same factors affect its development as affect mathematical development. These factors include specific numerical ones such as counting and general cognitive factors such memory functioning, processing speed, language and intelligence. Finally practice is suggested to be as relevant to explaining differences in mathematical development as it is for other areas of expertise.
Key words: Mathematic competence; procedural knowledge; conceptual knowledge; basic calculation.

Título: ¿Por qué los niños difieren en su competencia matemática en la escuela primaria?.
Resumen: Los niños difieren considerablemente en su competencia matemática en la escuela primaria. En este trabajo se revisan estudios que indican la importancia del status socioeconómico y la herencia antes de considerar el cálculo básico, la adición de un solo dígito y las sustracciones correspondientes. La destreza en cálculo básico varía con el dominio matemático más general. El retraso en el desarrollo de la destreza con el cálculo básico es común en los grupos de niños que avanzan lentamente en números, si también muestran retraso en la lectura. Alternativamente, el cálculo básico podría ser un indicador importante para el desarrollo matemático, ya que los mismos factores que afectan a su desarrollo también lo hacen con el desarrollo matemático. Estos factores incluyen habilidades numéricas específicas, tales como contar, y factores cognitivos tales como el funcionamiento de memoria, velocidad de procesamiento, el lenguaje y la inteligencia. Por último, se sugiere que la práctica debe ser lo más pertinente para explicar las diferencias en el desarrollo matemático como lo es para otras áreas en el aprendizaje.
Palabras clave: Competencia matemática; conocimiento procedimental; conocimiento conceptual; cálculo elemental.

In 2000, a British 10-year-old, Adam Spencer, achieved a B grade, a good pass, in Mathematics A-level. This is remarkable because A-levels are normally taken at the end of secondary school by 18-year-olds in the UK intending to progress to university. It is beyond the competence in mathematics required of UK primary teachers, which in turn is more than most of the population achieve by the time they finish secondary school. Although Adam's achievement is unusual enough to have made the news, marked diversity in primary aged pupils' attainment in mathematics is common enough to have been commented on in government reports (Cockcroft, 1982).

Understanding this diversity can inform policies and practices in primary education. Improving provision at primary level might even contribute to subsequent educational and economic success. Analysis of UK longitudinal data bases indicates some continuity between difficulties in primary school and difficulties in adulthood (Bynner & Parsons, 1997) and adults with numeracy difficulties in adulthood are at an economic disadvantage. For women with numeracy difficulties, these disadvantages are notable even when they have no literacy difficulties (Parsons & Bynner, 2005).

This review will not consider attempts to understand differences in national achievement (e.g. Stevenson *et al.*, 1990) or even diversity across classrooms (Mortimore, Sammons, Stoll, Lewis, & Ecob, 1988). This is because the differences between pupils in the same classroom seem substantially larger than any differences attributable to general culture or pedagogy. Instead the focus will be on factors that are supposed to explain differences between individuals, starting with background factors, and then moving to specific number skills and finally the role of general cognitive factors.

Socioeconomic status (SES)

Many studies show that family SES is associated with differences in children's educational achievement (Conger & Donnellan, 2007). Although researchers are still not agreed as how to measure SES, most studies use one or more of the following indicators: household income, parental education, and occupational status.

SES differences seem particularly acute when lower SES families are at extreme economic disadvantage. This has led some to emphasize the part played by limited economic resources in causing SES effects. Children in lower SES families are more likely to live in overcrowded accommodation with limited facilities. Poor health, malnutrition, and lack of sleep are likely to interfere with learning at school.

Another explanation of SES effects is that they reflect differences between families in parents' aspirations for their children and consequent investments of time and resources. Higher SES parents are likely to have progressed further in their own education. Their educational achievements are

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more likely to have enabled them to pursue their careers. They may be more determined that their children should also enjoy educational success. They may be better equipped to send their children to schools which value and deliver educational achievement.

Another path through which SES can affect educational achievement is through psychosocial adjustment. Children from lower SES families are more likely to show poor adjustment in school which will have its own effects on achievement.

Analysis of data from a longitudinal UK study indicated that at 7 and 11 years, material deprivation and parental involvement largely accounted for the relation between SES and achievement (Sacker, Schoon, & Bartley, 2002). They also accounted for variation in psychosocial adjustment which was independently related to educational achievement. The kind of school the child attended had increasing effect from 7 to 11.

Although SES is commonly considered to be an environmental variable, studies of the effects of SES on achievement in biological families confound environmental and genetic influences. This is because at least some of the variation in SES may reflect variation in parental genetic characteristics. Some of this will be shared by their offspring. One way of trying to get around this is to compare SES effects in adoptive and biological offspring. Adoptive families do not, however, represent the full range of SES: adoption agencies typically screen for income above a certain level. Johnson, McGue, & Iacono (2007) used a method of adjusting for restriction of SES range in their study of adoptive and biological families. They concluded that at least part of the SES association with educational achievement was environmental.

Genetics

One way to assess the contribution of genetic variation to explaining differences between people is to compare twins. Monozygotic (MZ) twins are genetically identical and dizygotic (DZ) twins are only as genetically similar as siblings who are not twins. On average, DZ twins will share 50% of the additive genetic variation and 25% of the dominance genetic variation (Plomin, DeFries, McClearn, & McGuffin, 2001).

Common and simple forms of estimating heritability only consider additive genetic effects. One method is to double the difference between the correlations for MZ and DZ pairs. Another method is to use the variance covariance matrices to estimate parameters for additive genetic influence (A), shared environmental influence (C), and non-shared environmental influence (E). This also yields confidence intervals for the parameters. Comparing the fit of models that include all three terms (A, C, and E) with ones that omit A or C is a way of identifying the most parsimonious model that can account for the data.

Although several studies have compared MZ and DZ twins' performance on mathematics tests the samples are usually small and so the confidence limits are broad. Also some samples featured children varying considerably in age. This meant that the abilities assessed varied and it ignored the possibility that genetic variation accounts for differing amounts of variation at different ages. A large cohort study such as the Twins Early Development Study (TEDS) can overcome these problems. TEDS consists of twins born in the United Kingdom from 1994 to 1996. It is a very substantial pool of families and children and has allowed large sample studies of the similarities between MZ twins and DZ twins.

Three studies of variation in primary mathematics have featured TEDS samples (Kovas, Harlaar, Petrill, & Plomin, 2005; Kovas, Petrill, & Plomin, 2007; Oliver *et al.*, 2004). Two used teachers' ratings of children's mathematics attainment at 7 years (Kovas *et al.*, 2005; Oliver *et al.*, 2004). Oliver *et al.* (2004) asked teachers to rate the children on three aspects of the primary maths curriculum. As the ratings were very highly correlated, I shall only describe the results for a composite derived from all three. The overall correlation between MZ twins was .74 and between DZ twins was .43, indicating heritability of 62 %. This was based on about 4000 children, about 1000 of each type of twin pair. A separate analysis of the children receiving the lowest 15% of ratings indicated similar results: .72 correlation between MZ twins, and .40 between DZ twins. Some twin pairs were in the same class and so rated by the same teacher. Others were rated by different teachers. This had a marked effect on the absolute level of correlation between ratings. MZ twin correlations were .83 for children in the same class and .57 when they were rated by different teachers. DZ twins in the same class correlated .51 but in different classes this dropped to .25.

Kovas *et al.* (2005) examined a slightly different sample of TEDS children to look at the relation between mathematics, reading, and intelligence. Teacher ratings of reading were supplemented by a reading test administered over the telephone. Intelligence was assessed over the phone too using subscales of an omnibus test. They concluded that most of the genetic variance in mathematics is common to that for reading and to a lesser extent, intelligence. This is broadly consistent with the generalist genes hypothesis (Plomin & Kovas, 2005).

Kovas *et al.* (2007) analysed data from about 1600 10-year-olds in the TEDS sample on web-based tests of five aspects of maths that were identified in the UK primary curriculum. The correlations between MZ twins on these varied from .51 to .63. Those for DZ ranged from .29 to .46. Estimates of heritability for the different aspects ranged from 34% to 48%. This is notably lower than the estimates derived from teachers' ratings in Oliver *et al.* (2004). This time the correlations differed little according to whether children were in the same classrooms (average MZ correlation .58, average DZ correlation .38) or different classrooms (average

MZ correlation .57, average DZ correlation .33). Analyses of the relations between aspects indicate that much of the genetic variation was shared, i.e. the same genes largely affected all five aspects. This is also consistent with the generalist genes hypothesis.

In discussing their results Kovas *et al.* (2007) take care to point out that substantial heritability does not mean that mathematical achievement cannot be changed. They resist the conclusion that differences in intelligence are responsible for the covariation of reading and mathematics on the grounds that what intelligence tests measure is itself little understood.

They also acknowledge that estimates are specific to samples and historical contexts. Even a sample as large as theirs is not sufficient to determine whether there are specific genetic factors for the different tests. Clearly genetic studies of this sort are extremely demanding in terms of sample size.

Basic calculation

Important as these studies of SES and genetics are, they do not take us very far in understanding variation. They rely on standardized assessments. Standardized assessments do not map onto any specific cognitive processes or strategies, even when these tests are differentiated according to curriculum relevant aspects (e.g. Kovas *et al.*, 2007). The lack of grounding in the psychology of number development makes variation in standardized maths tests as poorly understood as variation in performance on omnibus intelligence tests. It is possible that standardized tests inflate the effects of SES because they use story problems (calculation problems embedded in verbal contexts), which may place lower SES children at a particular disadvantage (Cooper & Dunne, 2000; Jordan, Huttenlocher, & Levine, 1992).

One candidate for explaining differences on maths tests for primary children is proficiency in basic calculation, the addition of single digit numbers and the corresponding subtractions. It shows substantial covariation with general arithmetic ability (Durand, Hulme, Larkin, & Snowling, 2005; Geary & Brown, 1991; Hecht, Torgesen, Wagner, & Rashotte, 2001; Siegler, 1988) and children selected for low attainment in mathematics consistently show deficiencies in basic calculation (Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Jordan, Hanich, & Kaplan, 2003; Landerl, Bevan, & Butterworth, 2004).

The course of development in basic calculation is well established and is the focus of a series of models increasing in sophistication (Shrager & Siegler, 1998; Siegler & Shipley, 1995; Siegler & Shrager, 1984). Children initially solve basic calculation problems such as “How much is 5 and 2?” by using retrieval of known combinations, guessing, or backup strategies involving counting. In the early years of primary school their knowledge of combinations increases and they develop more economical back up strategies, such as *min*, counting on from the larger addend, for addition, and, for subtraction, *counting down* from the subtrahend to the minuend or, less commonly, *counting up* from the minuend to the subtrahend (Thompson, 1999). They also make increasing use of *derived-fact* strategies, solving unknown combinations by using known combinations in conjunction with arithmetic principles, e.g. solving 8 - 4 by using the knowledge that 4 + 4 is 8 in conjunction with understanding the inverse relation between addition and subtraction, and *rules*, e.g. solving 5 - 5 by using inversion. Each strategy used increases in accuracy and speed (Siegler, 1987).

The main components of proficiency are accurate knowledge of combinations and mastery of efficient back up strategies. Knowledge of combinations enables solutions that are typically faster and more accurate than those using other strategies. However increasing reliance on retrieval only partly explains development in proficiency: back up strategy solutions become faster too.

Several studies have examined basic calculation characteristics informed by models of basic calculation development. For example Geary and Brown (1991) drew on Siegler and Shrager’s (1984) strategy choice model. In this model solution by retrieval is tried first. If no answer is retrieved that meets a specified confidence level the child resorts to a back up counting strategy. Answers to problems have different associative strengths. If the distribution is particularly peaked with one answer having much greater associative strength than others then that is the answer that will be given.

Geary and Brown (1991) compared a group of children identified as mathematically gifted with normal and mathematics disabled groups. All groups were about 10 years old though the gifted group were younger than the mathematics disabled group. Despite this the gifted group showed strategy characteristics resembling those of much older children and the mathematics disabled group resembled younger children. The gifted group relied on retrieval more often (gifted, 88%; normal, 61%; mathematics disabled, 44%). When they retrieved answers they were more accurate (gifted, 98%; normal, 89%; mathematics disabled, 91%). When they used a counting strategy they invariably used the more efficient *min* strategy and were faster. Indeed their estimated implicit counting rate was within the adult range.

Why might these differences arise? It might be that gifted children have more experience of solving problems. Through practice they increase speed and accuracy of back up strategy execution. By accurately executing back up strategies they increase the strength of association between correct solutions and associated problems. This then extends the range of problems for which retrieved answers can be relied on. Another consequence of practice is that they learn more from solving problems. For the child to learn from problem solving they must remember the problem and the solution. If by the time they have achieved a solution they have forgotten what the problem was then this is unlikely to be beneficial. Remembering both the problem and its solution is more likely with shorter solution times. Using more efficient counting strategies and counting faster enables shorter solution times.

Many studies have substantially replicated the characterisation provided by Geary and Brown (1991). An important extension is the demonstration of stylistic differences which can be interpreted in Siegler and Shrager's (1984) model as differences in confidence criteria. Siegler (1988) identified three groups in a sample of first grade children from middle income families by conducting a cluster analysis of their basic calculation characteristics: good students, perfectionists, and not-so-good students. Good students were faster than not-so-good students when they used retrieval and when they used back up strategies. They were also more accurate on both strategy types and used retrieval more often. Perfectionists were faster and more accurate than not- so-good students on both strategy types but they used retrieval less often than the other groups. Perfectionists and good students did not differ on achievement measures. Both groups outperformed the not- so- good students. Perfectionists can be seen as setting higher confidence criteria for relying on retrieved answers. The three-way classification has been replicated in two further studies.

Kerkman and Siegler (1993) retested a group of children from lower-income families after six months. They found that all children initially classified as good students remained good, most perfectionists were still perfectionists but some had migrated to the good category. The not-so-good students were mixed: some were now good, others perfectionists and some were still not-so-good. Consistent with the model, initial accuracy on back up strategies predicted later retrieval frequency and accuracy.

Kerkman and Siegler (1997) reported data from a larger sample of children from both lower and middle income families. Once again the perfectionists and good students did not differ on overall mathematics achievement measures and both scored higher than the not-so-good students. Again the perfectionists' infrequent use of retrieval seemed due to caution rather than lack of knowledge.

More recent models of strategy development have increased their scope and power. The latest model (Siegler & Shrager, 1998) incorporates strategy discovery as well as features that learn about strategies and problem types, not just the specific problems solved. The strategies that children seem to invent for themselves are legitimate and conform to a goal sketch. A goal sketch represents conceptual knowledge at some level which may not be verbalisable or amenable to introspection. Siegler and Crowley (1994) explored preschool children's judgments of strategy use as a means of assessing goal sketches. Children who could add but did not use *min* judged *min* strategy use to be smarter than quick illegitimate strategies and tended to judge *min* to be smarter than their own strategy. This method of assessing conceptual knowledge is quite different from the ways other researchers have used. They have been more concerned with assessing children's knowledge of arithmetical principles.

All the economical counting strategies implicitly assume arithmetical principles. The use of *min*, counting on from the larger addend even when it is not the first addend, implicitly assumes commutativity- the irrelevance of addend order to the sum, *i.e.* '$a + b = c$' *implies* '$b + a = c$'. The use of *counting up* to solve a subtraction problem, e.g. solving 6 -4 by counting up from 4 to 6, implicitly assumes the inverse relation between addition and subtraction *i.e.* '$a + b = c$' *implies* '$c - a = b$'. Counting down to solve a subtraction, e.g. solving 6 – 4 by counting down from 6 to 4, implicitly assumes the complementarity of subtraction, i.e. '$c - b = a$' implies '$c - a = b$'. Researchers have used a variety of methods to assess children's knowledge of these principles. One method is to embed pairs of conceptually related problems in a set of calculation problems and leave it to the child to point out the connection (Russell & Ginsburg, 1984). Another is to present children with the answer to a large number problem that is either beyond their ability to solve (Dowker, 1998) or beyond their ability to solve rapidly (Jordan *et al.*, 2003) and then ask them to solve a related problem. Another way is to ask children to judge whether a puppet could use a solved problem to work out the answer to another (Canobi, 2004).

Although some research reports associations between knowledge of arithmetic principles and strategy characteristics (Canobi, 2004, 2005; Cowan & Renton, 1996) and children with mathematics difficulties show deficits in both (Jordan *et al.*, 2003), the evidence base is limited: only some aspects of conceptual knowledge have been studied, and the nature of the relationship is unclear. Conceptual knowledge and strategies could be related in one of three ways: conceptual knowledge may lead to strategy development, development of strategies may enhance understanding of principles, or developments in either may lead to developments in the other (Rittle-Johnson, Siegler, & Alibali, 2001).

As the evidence of an association is based on correlations rather than interventions, the link between strategy and principle could also be due to other factors. These other factors may be specific number factors or more general cognitive factors.

A plausible candidate for a specific number factor is counting.

Counting

Proficient counting may be relevant to basic calculation in several ways. Firstly, common back up strategies involve counting. More fluent knowledge of the count sequence will facilitate more accurate and rapid execution of strategies, as well as enhancing the probability of remembering the problem and the solution. The shift from overt counting to silent counting is an important step forward: recall the adult-like implicit counting rates of the gifted students in Geary & Brown (1991). Deficits in counting fluency have been observed in children with arithmetic difficulties (Geary, 1990; Hitch & McAuley, 1991; Landerl *et al.*, 2004) and speed of counting varies with more general measures of arithmetic in children showing average and above average attainment (LeFevre *et al.*, 2006).

Secondly, more efficient counting strategies involve counting on or down from specific numbers. When children first learn to count, they are unable to do this. In the 'unbreakable chain' phase (Fuson, Richards, & Briars, 1982) children always start counting from one and seem unable to continue from other points. This would limit them to the most laborious of counting strategies. Although the basis for the development of more flexible counting is contested (Fuson, 1988), the consequences for calculation are clear. Our previous work (Donlan, Cowan, Newton, & Lloyd, 2007) found inaccuracies in counting on and down were strongly associated with errors in basic calculation.

Thirdly, through counting children master the number word sequence. There is a lag between being able to recite the number word sequence and having insight into the numerical relationships it embodies. For example, Siegler & Robinson (1982) found knowledge of the relative magnitude of single digit numbers seems to follow counting to ten by about two years. Knowledge of relative magnitude might be developed through knowledge of the count sequence, i.e. learning that numbers that come later in the count sequence are larger than earlier numbers, though it may be further enhanced through the development of a central structure for whole numbers that includes a mental number line (Griffin, Case, & Siegler, 1994).

In adults and older children, there is evidence for some basis for relative magnitude other than counting knowledge: adults and older children show a distance effect. They are faster to judge relative magnitude the greater the difference between the numbers, e.g. they are quicker to judge that eight is more than four than to judge that four is more than three. If going through the number word sequence was the basis for relative magnitude judgments then faster times would be expected for closer numbers.

Knowledge of relative magnitude is presupposed in the *min* strategy for addition and implicated in retrieval of combinations (Butterworth, Zorzi, Girelli, & Jonckheere, 2001). Single digit magnitude comparison speed associates strongly with basic calculation proficiency (Durand *et al.*, 2005) and children with arithmetic difficulties are slower to compare magnitudes (Landerl *et al.*, 2004).

Another way in which counting development may support development of basic calculation is through enhancing appreciation of number patterns. These may play a role in developing strategies and knowledge of combinations (Baroody, 1999). Problems such as $n \pm 1$, $1 + n$, and $n - (n - 1)$ can be seen as embodying simple rules. Appreciating the $n + 1$ and $1 + n$ rules may play a part in developing the *min* strategy.

Finally proficient counting involves understanding counting principles. Understanding of counting principles, as indexed by discrimination of erroneous counts from orthodox and unconventional legitimate counts, also varies with arithmetical proficiency (Geary *et al.*, 2004; LeFevre *et al.*, 2006).

In a study tracking children from the beginning of kindergarten, Jordan, Kaplan, Locuniak, and Ramineni (2007) administered a number sense battery six times and used a standardized test to assess maths achievement after about 18 months. The battery included a subscale assessing counting skill and principles and another assessing number knowledge. The latter included items assessing number sequence knowledge and relative magnitude. Both predicted variation in maths achievement but the correlations with number knowledge were greater. The number knowledge correlations at different time points with mathematics achievement ranged from .52 to .57. The correlations with the counting subtest varied between .28 and .37. The whole number sense battery was even more successful in predicting variation in achievement, correlations ranging from .66 to .73. The other subscales that contributed to the battery assessed knowledge of number combinations, ability to solve story problems, and performance on nonverbally presented calculation problems.

The importance of the skills captured by the number sense battery is indicated both by the overall strength of the relationship and the finding that variables such as family income, gender and reading ability did not explain any more of the variation in achievement. Even though it is a predictive relationship it is still just a correlation and so it is possible that any of the relationships between number sense, counting proficiency, basic calculation and achievement are due at least in part to other factors that influence them all. Such factors include general cognitive factors.

General cognitive factors

Reading and mathematics achievement covary substantially. Most children who have difficulties with mathematics have difficulties with reading too. Although some children with reading difficulties are unimpaired in arithmetic, and some children with arithmetical difficulties have no reading difficulty, children who show low attainment in both have greater impairments in number than those with just impaired number development (Jordan *et al.*, 2003).

This indicates the importance of factors common to the development of both. Candidate cognitive factors include working memory, processing speed, language skills, and intelligence.

Working memory is a very plausible candidate as it has connections with learning and is involved to varying degrees in most cognitive tasks, such as listening, reading, reasoning, and arithmetic. The outline model proposed by Baddeley and Hitch (1974) has provided a framework for research. This model consists of three components: the phonological loop, the visuospatial sketchpad, and the central executive. The functioning of all components improves with age (Gathercole, 1998) and variation in working memory tasks correlates with educational achievement (Gathercole & Pickering, 2000a, 2000b; Hitch, Towse, & Hutton, 2001).

The phonological loop is a temporary storage system for sounds. Its level of functioning is assessed by tasks in which the child reproduces an arbitrary set of words, nonwords, or numbers in the same order as presented. The forward span is the length of the longest sequence correctly reproduced on most trials. Spans relate to variation in reading. Phonological loop functioning affects vocabulary acquisition (Jarrold, Baddeley, Hewes, Leeke, & Phillips, 2004). Although some have suggested a role for the phonological loop in counting and calculation, the evidence of variation in phonological loop functioning with mathematics achievement is mixed. Some studies have reported children with mathematical disabilities have shorter forward spans than typically developing children (e.g. Geary, Brown, & Samaranayake, 1991), even when the groups are matched on reading ability (Hitch & McAuley, 1991), but others have not (Geary, Hoard, & Hamson, 1999).

The visuospatial sketchpad is a temporary storage system for visual and spatial material. The Corsi blocks task is one method of assessing its functioning. In this task the child sees a board with a set of identical blocks haphazardly arranged on it. A number of blocks are touched in a particular order and the child's task is to touch the same blocks in the same order. The Corsi span is the longest sequence reliably reproduced. Roles have been proposed for the visuospatial sketchpad in mental and written arithmetic, and in solving story problems. Baddeley (2003) suggests its role in reading may be in keeping track of position on the page. If children learnt combinations from tables of facts then it would be involved in learning them. Some studies find differences in Corsi span between mathematics difficulty and typically developing groups (e.g. McLean & Hitch, 1999), others do not (e.g. Bull, Johnston, & Roy, 1999).

The central executive is involved in control of attention, switching retrieval strategies, and activating information in long-term memory. Tasks that assess it involve both processing and storage of information. The simplest test is backward digit span –recalling a set of digits in the opposite order to that heard. Other tests include listening span where a child must make judgments about a series of sentences and subsequently recall the last words of each sentence, and counting span, in which the child must count the dots on each of a set of pictures and subsequently recall the number on each picture. Of all the working memory components, central executive assessments tend to correlate the highest with variation on number tasks (e.g. Cowan, Donlan, Newton, & Lloyd, 2005), though this can vary with group (Henry & MacLean, 2003). Using other measures of executive functioning, some studies have been able to account for more variation (Bull *et al.*, 1999; Bull & Scerif, 2001).

If working memory functioning is important for the development of both reading and number then one might not expect to find much covariation between measures of working memory and number skills after controlling for reading achievement. Although Bull and Johnston (1997) did find some, its contribution was eliminated when two other measures were included. These other measures were of speed: rapid automatized naming (RAN) and processing speed.

Performance on RAN tasks, which require children to name familiar items such as letters or numbers as fast as they can, has emerged as one of the best predictors of reading (Powell, Stainthorp, Stuart, Garwood, & Quinlan, 2007). Although some consider RAN tasks just assess an aspect of phonological processing (e.g. Hecht *et al.*, 2001), Powell *et al.* found reading impairment in children who showed RAN without any accompanying phonological deficits. The models that best accounted for the data in their large scale study of 7- to 10- year-olds treated RAN as separate from phonological awareness and phonological memory.

One can construe RAN tasks as assessing general efficiency in retrieving information from long term memory. If that is what they do, then variation in RAN might account for some variation in constituents of basic calculation such as retrieval of number facts and counting fluency. Hecht *et al.* (2001) found RAN measures uniquely accounted for growth in mathematics achievement in the early grades of primary school even when reading skills were controlled.

A different view of RAN tasks is that variation in them reflects differences in general processing speed (e.g. Kail, 1991). Although speed of counting and strategy execution have been mentioned above as contributing to variation in calculation, differences in these speeds can be explained in two ways. One is that slower speeds result from less experience: after all even adults increase their speed with practice on most cognitive tasks. This is the idea one source of variation in speed is task specific. The other explanation is that there are general differences in processing speed that may account for variation in speed between older and younger children independently of task specific knowledge and practice. Whereas Bull and Johnston (1997) found general processing speed was the strongest predictor of arithmetic proficiency and explained away the associations with working memory and RAN, others have not: Hitch, Towse, and Hutton (2001) found working memory to be a stronger predictor than processing speed. Durand *et al.* (2005) did not find any independent contribution of processing speed though it was related to single digit magnitude comparison speed which was important. General processing speed did not account for the relation between RAN and reading in the study by Powell *et al.* (2007).

Variation in language skills is likely to account for differences in children's number skills and general maths achievement as well as reading. Counting and in particular mastering the number word sequence draws on a child's linguistic resources. Story problems make demands on children's language comprehension, sociocultural knowledge, and verbal reasoning.

Children's linguistic resources are also likely to influence their ability to benefit from instruction at home and at school, whether it is in understanding what people say, contributing to discussion, or asking questions. A problem for researchers trying to assess the importance of linguistic skill is that many measures of it are considered to be measures of intelligence. So are the associations between verbal measures and number skills a reflection of the importance of language or the importance of intelligence?

Children with specific language impairment (SLI) might seem to be a natural group to separate the effects of language impairment from deficits in intelligence: by definition they show oral linguistic impairments in combination with intelligence in the normal range, as assessed by nonverbal reasoning tests such as Raven's Coloured Progressive Matrices (Raven, Raven, & Court, 1998). The classic profile of a child with SLI is to show deficits in both phonological processing and understanding of language. They are known to have a substantial risk of reading difficulty. In the US approximately 40% of children with SLI will meet the criteria for reading difficulties (Catts, Fey, Tomblin, & Zhang, 2002). In contrast only 8% of typically developing children with similar nonverbal ability will do so.

Our recent studies compared three groups of children: an SLI group aged between 7 and 9 years, an age control group (AC) matched for chronological age and nonverbal reasoning, and a language control group (LC) matched with the SLI group on language comprehension (Cowan *et al.*, 2005, in press; Donlan *et al.*, 2007) and age-adjusted nonverbal reasoning. The LC group were on average two years younger than the other groups. Comparisons of the children's knowledge of the number word sequence and number combinations, of their basic calculation and story problem proficiency, and their ability to seriate and use ordinal number, all showed the SLI group to be indistinguishable from the much younger LC group and very much below the level of their peers, the AC group. On most other tasks SLI group performance fell between the AC and LC groups. Only on a test of commutativity using arbitrary symbols did the SLI group resemble the AC group.

Striking as the differences were, they are not unequivocal evidence of the importance of language skills: the SLI group also differed from the AC group in working memory. We conducted multiple regression analyses to control statistically for other characteristics. These analyses suggested that language comprehension, nonverbal reasoning, and working memory functioning varied in their importance for different number skills but the amounts of variance uniquely accounted for were small even though overall variance accounted for was moderate to large (R^2s between .31 to .75). This is because the different factors correlate with each other.

More generally, the relationships between different cognitive factors make it difficult to discriminate between them empirically. Studies that leave some factors out risk inflating the importance of those included. Even if all factors were included, the confidence limits for individual coefficients are likely to be wide due to common sample sizes which fall far short of those in cohort studies.

Another issue is that in theory too, cognitive factors are interrelated. Working memory functioning should affect performance on tests of language, processing speed, and reasoning. Conversely linguistic knowledge and processing speed are claimed to influence performance on working memory tests. Working memory, language, and speed of information processing have formed subscales of omnibus intelligence tests. Just what 'purer' intelligence tests such as Raven's assess that is separate from working memory is disputed (Ackerman, Beier, & Boyle, 2005; Colom, Rebollo, Palacios, Juan-Espinosa, & Kyllonen, 2004; Engle, Tuholski, Laughlin, & Conway, 1999).

In conclusion

The relation between basic calculation and more general proficiency might be explained in several plausible ways. Proficiency in basic calculation may provide an essential platform for further arithmetic. As most of current primary mathematics is arithmetic, children who are insecure in basic calculation are ill equipped to progress. That may be relevant for understanding below average progress but it does not seem very likely to be the answer for explaining differences between children making above average progress and more typical children.

The covariation with other factors explanation is also plausible: factors that cause children to differ in basic calculation development will also cause them to differ in more general mathematical development. General cognitive factors have better prospects for accounting for variation across the range but it should be remembered that the amount of variation in mathematics or simple calculation that has been accounted for is limited. This may be because of defects in measurement quality or omission of relevant variables.

One relevant variable that is typically omitted is experience. By default amount of experience is assumed to be equal but this is unlikely to be true. Differences in experience in the form of practice is likely to play a major part in explaining differences in children's achievement in mathematics, just as it does in adult expertise (Ericsson, Roring, & Nandagopal, 2007). As Adam Spencer said when being interviewed about his A-level success "I don't live a completely strict life, but I do work each day."

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(Article received: 4-4-2008; accepted: 15-5-2008)