The Post-Stagnation Stage for Mature Tourism Areas: a Mathematical Modelling Process[*]

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Keywords: Tourism Area Life-Cycle model, innovation, logistic growth**.**

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Abstract:

The Tourism Area Life Cycle (TALC) model by Butler (1980) explains the temporal evolution of a tourism resort. Lundtorp and Wanhill (2001) find that the logistic growth model represents the first phases of the TALC model. However, since the logistic model assumes a fixed tourism market ceiling, it fails to explain the post-stagnation stage, where rejuvenation, decline or any other intermediate possibility may arise. Taking into account the data of passenger flows to Bornholm from 1912 to 2001 collected by Lundtorp and Wanhill, we find that the superposition of several logistic growth models fits better with these data. Then, we propose a multi-logistic growth model, where the investment or innovation in the tourism sector boosts the addition of new logistic curves which superpose the old ones. The continuous birth and superposition of these new life cycles is not free, it requires purposive effort of entrepreneurs and governments seeking new markets and the improvement of infrastructures

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Introduction

The evolution of tourist destinations has been theoretically and empirically represented by the tourism area life cycle (TALC) model following the ideas put forward by Butler (1980). The model proposes an S-shaped growth pattern for a tourist destination, with several key phases, from exploration to stagnation. Once the upper limit of the curve is achieved, a decline, rejuvenation or other intermediate solutions are possible for this destination. The characteristics of a poststagnation phase have been debated in the literature (Butler, 1980; Debbage, 1990; Argawal, 1997; Priestley and Mundet, 1998). Some authors (Agarwal, 2002, 2005) claim that all destinations face an irremediable decline. Others, like Aguiló et al (2005), analyzing the Balearic Islands, and Claver et al. (2007), evaluating Benidorm, found that some mature destinations are going through rejuvenation processes.

The rejuvenation process is the result of a change. Destinations which are in the post stagnation stage, normally identified as mature destinations, decline if obsolescence is not counteracted, but they can rejuvenate if their supply reacts to the saturation and the new needs and wishes of the demand (Hernández and León, 2007). Many of these destinations, usually by building on original attractions such as beaches, scenery, culture or climate (Butler, 2009), have grown both in number and in a more sophisticated supply, in quality of infrastructures and facilities, in a greater range of offerings of attractions, and with cheaper, quicker and easier access from other regions (Butler, 2011).

The ability of destinations to absorb tourists is being modified due to this expansion of the supply. The goods and services offered by the destination define and change the maximum number of tourists that could be accommodated by a destination. The wider and more varied number of services and attractions, the greater the number of tourists that can be received simultaneously in the same geographical space, enjoying several goods and services simultaneously (Aguiló, Alegre and Sard, 2005). Furthermore, if the goods and attractions offered by a destination vary or change, or if the quality of their infrastructures and facilities increases, the limits of growth of the destination are modified (Albaladejo and Martínez-García, 2014). Additionally, technical progress in transport and infrastructures may provide a better explanation of its evolution, as it is proved by Kato and Mark (2013) in Hawaii. Hence, the market ceiling of a tourist destination is subject to change.

Destinations can expand their potential simply by rejuvenating the products and services, by investing in developing new ones, by opening up to new markets, by improving the communication infrastructures, etc. These activities require purposive efforts by entrepreneurs and governments. However, the possibilities are not bounded. Investment and inventiveness could be the driving force of a continuous birth of new life cycles which superposes the old ones. The sum of all of these continuously enhances the tourism market ceiling, giving a chance for unbounded growth. When these efforts are not applied, stagnation is the outcome in the best of the scenarios. Sometimes, overcrowding and depreciation of services and infrastructures could lead to a decline.

The aim of this paper is to show that the widely accepted mathematical model by Lundtorp and Wanhill (2001) can be extended by introducing a dynamic market ceiling and this extension can be mathematically formulated using an increasing multi-logistic growth model. Our mathematical formulation allows increases in the ceiling, as a result of purposive efforts by entrepreneurs and/or governments. If no effort is made, obsolescence and depreciation can drive a decline period.

The paper is organized as follows. Section 2 reports evidence on the superposition of several life cycles in Bornholm (1912-2014). We used the data disclosed by Lundtorp and Wanhill (2006) up to 2001. The entire series was later given to us by Wanhill.[1] Section 3 proposes a multilogistic growth model as a good approximation of the evolution of a touristic destination. Investments on infrastructures and efforts to innovate boost a process of continuous birth of new life cycles (logistic growth pattern) which superpose the old ones. The sum of all of them unboundedly enhances the tourism market ceiling. Section 4 provides a mathematical model (two-differential equations system) that represents all the phases of the Tourism area life cycle model, including the post-stagnation phase with its different possibilities: stagnation, decline, rejuvenation or other intermediate solutions. Section 5 concludes.

Multi-logistic growth in Bornholm

The TALC model argues for the existence of an S-shaped lifecycle in the growth of the destinations with six key phases: exploration, involvement, development, consolidation, stagnation, and decline and/or rejuvenation. The following diagram by Butler (1980) represents these stages:

FIGURE 1

Lundtorp and Wanhill (2001) find that this sinusoidal development of a tourist destination can be theoretically approximated by a logistic growth model. Using this conceptual framework, these authors explain the processes that may generate the different phases in the development of a resort, although it may or may not fit any particular case.

The logistic growth model, first proposed by Verhulst in 1838 as a population model (see Clark 1990), says that

$$\dot{T}(t) = \alpha T(t)\left(1 - \frac{T(t)}{C}\right); \quad T(0) = \overline{T} > 0 \tag{1}$$

where T is the number of visitors (tourists)[2], $\dot{T}$ is its temporal derivative, $\alpha > 0$ is a parameter that expresses the speed of expansion of the number of tourists at the destination and C is the upper limit, called demand ceiling.

As is known, the solution of the differential equation (1) is

$$T(t) = \frac{C}{1 - e^{-\alpha(t-t_0)}} \qquad \text{with } t_0 = \frac{1}{\alpha} ln\left(\frac{C - \overline{T}}{\overline{T}}\right), \tag{2}$$

which is a sinusoidal curve with a turning point at $t = t_0$. As the number of tourists T approaches the demand ceiling C the growth vanishes. The literature defines this concept as the specific level of acceptance of tourist development and use, beyond which further development can cause destruction of the physical, economic and sociocultural environment or a decline in the quality of the visitors' satisfaction (Saveriades, 2000). A considerable number of definitions in terms of limits of economic, social and physical capacities of the destination, often expressed in numbers of tourists per unit of time or density, have been used to analytically determine this concept (Cole, 2009).

Function (2) is a quite good theoretical representation of the first five stages established by the TALC theory (exploration, involvement, development, consolidation and stagnation), see Figure 1. Then, if a logistic curve fits to the data (arrivals, accommodation or receipts), the slope of the curve at any particular period of time would identify the phase of TALC where the destination is situated. Lundtorp and Wanhill (2001 and 2006) found that, in Bornholm, the logistic curve depicted in Figure 2 (T_1) fits to the data of the number of tourists from 1912 to 1967.[3] Bornholm is a Danish island in the Baltic Sea next to Copenhagen and reached by ferry. This curve fitted by Lundtorp and Wanhill was a maximum likelihood estimation of a fragment of the data. They only used data up to 1967 because, as the authors argued, many of the observations from 1968 onwards differed from the trend and although a polynomial could be used to fit the data, it would not represent the typical Butler curve.

FIGURE 2

Figure 2, by Lundtorp and Wanhill (2006), plots the observations from 1912 to 2001 against the estimated logistic function. It is clear that there are periods of growth and stagnation for Bornholm that are not represented by the estimated logistic function T_1. Note that, according to this function, the stagnation stage in Butler's theory should start in 1980 when the number of tourists arriving in Bornholm is increasing.

In their data analysis, Lundtorp and Wanhill (2001, 2006) explain that the change in the life cycle of Bornholm was due to growth in alternative markets for the island. Thus the Butler curve fits best whenthere is a dominate market of repeat vistors, as in the case of the sales of a single product at the micro level. This was the case for Bornholm up until the 1960s when it was predominently a Danish holidy island. What is observed in Figure 2 is the Danish market falling away with the growth of overseas package holidays. If an alternative market is introduced into the life cycle of a destination during a period of logistic growth, a second period of logistic growth can superimpose on the first growth pulse (Meyer, 1994), and so on. So, we think that the time path of Bornholm can be better represented with a model with two or more logistic growth pulses, growing at the same time or sequentially.

In order to test this idea, three functions with two (T_2), three (T_3) and four phases (T_4) of logistic growth were fitted to the data of the number of ferry passengers to Bornholm from

1912 to 2014. This was done using non-linear least squares regression with the nmlrt package of R (Nash, 2012). The data from 1912 to 2001 had already been used and disclosed by Lundtorp and Wanhill (2006). The entire series was given to us by Wanhill.

The results for the functions estimated T_2 and T_3 are summarized in Tables 1 and 2. In Figures 3 and 4 the observations are plotted against the estimated functions T_2 and T_3, respectively.

IMAGE 1 (Tables 1 and 2 and Figures 3 and 4)

Based on the taxonomy proposed in Meyer (1994), the curves with two (T_2) and three logistic phases (T_3) in Figures 3 and 4 are said to be sequential. A phase does not start growing until the previous phase has reached about 98% of its market ceiling in both cases. This can be better observed in Figures 5 and 6, where each pulse is drawn independently.

IMAGE 2 (Figures 5 and 6)

Lundtorp and Wanhill (2001, 2006) explain that there were three growth phases in Bornholm up to 2001. A first period was defined by the historical tourists that Bornholm received until 1951, excluding data from World War II. From 1953 until 1967, the number of tourists arriving in Bornholm increased steadily. They were primarily Danish as noted earlier. From 1976, the demand pattern changed with tourists arriving from Germany and Sweden. These new markets were the result of widening Bornholm’s market appeal as a resort destination, the gradual expansion of arts and crafts products for which the island now has a worldwide reputation, and the opening of new ferry routes. Recently, a major investment in infrastructure has facilitated travel to Bornholm, with the official opening of the Oresund Bridge in 2000, connecting Copenhagen and southern Sweden (Ystad) by road, and then a fast ferry route to Bornholm. This made Bornholm a short, as well as a long, holiday destination for its neighbouring markets. Assuming the existence of these four periods, we propose a four-phase logistic growth model, defined by the following expression:

$$T_4(t) = \frac{C_1}{1+e^{-(\alpha t-\beta_1)}} + \frac{C_2}{1+e^{-(\alpha t-\beta_2)}} + \frac{C_3}{1+e^{-(\alpha t-\beta_3)}} + \frac{C_4}{1+e^{-(\alpha t-\beta_4)}}$$

The results obtained for the estimation of this function are summarized in Table 3 and depicted in Figure 7.

IMAGE 3 (Table 3 and Figure 7)

The estimated curve with four phases in Figure 7 also follows a sequential process for the first three pulses, but the last begins when the previous one is still increasing, as can be seen in Figure 8.

FIGURE 8

The maximum value of the growth rate to the first logistic curve is reached in 1934. From this year, the growth rate starts to fall. Around 1950, the first curve reaches 99,5% of its market ceiling before the second curve begins to grow. For this second curve, the maximum value of its growth rate is reached at the end of the 50´s. The third curve begins to grow when 99,7% of the second curve ceiling is saturated. This third curve reaches its maximum growth rate in 1987. The fourth curve starts to grow when the previous one has only saturated 93% of its ceiling. The maximum growth rate for this curve is obtained at the begining of the new century (around the year 2002).

Graphically, the functions T_3 and T_4 seem to fit the data better than T_1 and T_2. Moreover, using Akaike's Information Criterion (AIC) and the Bayesian information criterion, or Schwarz criterion (BIC), the results of which are presented in Table 4, functions T_3 and T_4 are revealed the best estimations. However, the AIC says that T_4 is the best one while the BIC supports the T_3 function.

TABLE 4

The following step in studying the evolution of touristic areas is to identify the causes that make this evolution follow a single logistic growth model (with an early stagnation phase) or a multi-logistic one (where stagnation is postponed). In the following section we propose innovation as the driving force behind the birth of new life-cycles.

The Multilogistic growth model

We have just proved that the overlapping of several life cycles could explain the touristic development in Bornholm. As Lundtorp and Wanhill (2001, 2006) explained, the birth of a new cycle responds to a change in tourists preferences or to an improvement in market access. That is, a new cycle could arise as a consequence of an investment or an innovation.

Throughout history, tourism has been characterized by immense innovativeness. According to Hjalager (2010), there has been innovation in service or product, that is, changes directly observed by the customer and regarded as new (never seen before at a particular destination). However, some innovations, although not directly observed, can enhance the business volume by enhancing the efficiency (process innovation) or by promoting staff and improving their labor conditions (managerial innovation). Marketing innovativeness can also help to promote certain destinations and improve networks and alliances (institutional innovations). Each of these innovations contributes to the rejuvenation of a particular tourism area. As an example, Spain and the Mediterranean destinations, typical seaside tourism destinations, are increasing their supply with other forms of tourism such as cultural, health, religious, or rural tourism. Several types of tourism overlap at the same destination and they are not mutually exclusive, because in many cases tourists can enjoy several experiences simultaneously (González and Bello, 2002). Any of these breakthroughs could open a new cycle, which is added to the previous one, as depicted in Figure 8. The single lifecycle model follows the famous S-curve. A tourism area develops along overlapping S-curves. In general, the addition of N life-cycles at a specific destination could be represented by a multi-logistic model, that is:

$$T_N(t) = L_1(t) + L_2(t) + \ldots + L_N(t) = \frac{C_1}{1+e^{-\alpha(t-t_1)}} + \ldots + \frac{C_N}{1+e^{-\alpha(t-t_N)}} \tag{3}$$

where L_i represents the number of tourists arriving the country as a consequence of the $i-th$ innovation (in any of its categories: product, process, management, marketing and institutional innovation).

Note that, for each i, L_i satisfies the logistic growth differential equation (1) for a certain level of demand ceiling C_i. That is, a specific breakthrough (a new product or service for example) experiences the six phases explained by the theory (exploration, involvement, development, consolidation and stagnation) and the stagnation arises when the growth limit

C_i is reached. However, there is no finite limit for the destination if new lifecycles emerge continuously. As Stankey and Schreyer (1985) argue, many possible limits exist at a destination, depending on each form of tourism (sports tourism, cultural tourism,...). The rejuvenation of a tourist destination occurs if a more sophisticated supply is offered, in terms of a greater range of offerings of attractions (Butler,2011).

Innovation could spur new future life-cycles. The number of cycles, N, could be increased by investing in R&D oriented tourism. This investment can generate a temporal evolution for variable $N(t)$ and then

$$T_N(t) = \sum_{1}^{N(t)} L_j(t) = \sum_{1}^{N(t)} \frac{C_j}{1+e^{-\alpha(t-t_j)}}. \tag{4}$$

Note that the available forms of tourism $0 \le j \le N(t)$ will evolve with time t. The accumulated demand ceiling will be $K(t) = \sum_{1}^{N(t)} C_j$ and will evolve with time if $N(t)$ increases.

A mathematical model for a comprehensive modelization of TALC Theory

Having a closed form differential equation like (1) has been proved to be useful in many tourism studies (Lundtorp and Wanhill, 2001 and 2006; Cole, 2009 and 2012; Albaladejo and Martínez-García, 2014). For this reason, is desirable to find a closed form differential equation (or equation system) also for the multi-logistic model. This is the aim of this section, which uses a more technical mathematical language. The technical argumentations can be skipped and the reader can go directly to the final result (Proposition 2). The differential equation system (8)-(9) describes the development of a tourist destination which devotes efforts to tourism innovation, so promoting the continuous superposition of life cycles.

The following proposition proves that the addition of several logistic curves can be expressed as a single logistic curve with a ceiling which is the aggregate of all the single ceiling, that is,

Proposition 1 *Given the addition of* N *logistic curves with turning points* $t_1, t_2, \ldots$. t_N *(increasingly ordered) there always exists a date* s, $t_1 \le s \le t_N$, *such that*

$$T_N(t) = \sum_1^N L_j(t) = \sum_1^N \frac{C_j}{1+e^{-\alpha(t-t_j)}} = \frac{K}{1+e^{-\alpha(t-s)}} \qquad (5)$$

with $K = \sum_1^N C_j$ the accumulated demand ceiling. At time t=s half of the accumulatd ceiling K is reached.

Proof. We shall start by assuming that $N = 2$. For this case let

$$f(x) = \frac{C_1 + C_2}{1+e^{-\alpha(t-x)}}$$

be a continuous function for $x \in \mathrm{R}$. Note that if $t_1 \le t_2$ then $f(t_1) \ge f(t_2)$, that is

$$\frac{C_1 + C_2}{1+e^{-\alpha(t-t_1)}} \ge \frac{C_1 + C_2}{1+e^{-\alpha(t-t_2)}}.$$

Note also that

$$f(t_1) \ge \frac{C_1}{1+e^{-\alpha(t-t_1)}} + \frac{C_2}{1+e^{-\alpha(t-t_2)}} \ge f(t_2).$$

Since $f(x)$ is a continuous function there exists a unique value s such that $t_1 \le s \le t_2$ and

$$f(s) = \frac{C_1}{1+e^{-\alpha(t-t_1)}} + \frac{C_2}{1+e^{-\alpha(t-t_2)}}.$$

That is

$$\frac{C_1}{1+e^{-\alpha(t-t_1)}} + \frac{C_2}{1+e^{-\alpha(t-t_2)}} = \frac{C_1+C_2}{1+e^{-\alpha(t-s)}}.$$

For $N > 2$ we proceed by induction.■

At any instant of time t the number of coexisting cycles is $N(t)$. This variable should be viewed as a tractable proxy for the complexity of the tourism destination or alternatively for the average degree of diversity. This broader notion of $N(t)$ would be continuous rather than discrete. We could justify the continuous nature of N formally by shifting from the sum over a discrete number of types in equation (5) to an integral over a continuum of types:

$$T(t) = \frac{\int_0^{N(t)} C(j)dj}{1+e^{-\alpha(t-s(t))}} = \frac{K(t)}{1+e^{-\alpha(t-s(t))}} \qquad (6)$$

where $K(t)$ is understood as the accumulated demand ceiling and $s(t)$ is the instant in time where half of the accumulated ceiling is reached. Note that time differentiating $K(t)$,

$$\dot{K}(t) = C(N(t)) \cdot \dot{N}(t), \qquad (7)$$

that is, the accumulated demand ceiling will increase if and only if the diversity $N(t)$ increases. Equation (7) says that an increment in the ceiling $K(t)$ will be equal to the ceiling of the latest breakthrough times $\dot{N}$, the increment in diversity. In the simplest case, if the ceiling is constant $C(N) = \overline{C}$ for all N, we have that $K(t) = \overline{C}(N(t) - N_0) + K_0 = \overline{C}N(t)$, where N_0 and K_0 are the initial values of N and K respectively ($K_0 = \overline{C}N_0$).

According to the Schumpeterian approach, entrepreneurs provide the major contribution to innovative dynamics. Hjalager (2010) also recognizes environmental factors such us market changes and political issues as driving forces of innovation. Innovation literature assumes that the innovative capacity of the human being is unlimited; however, any advance requires effort in the form of R&D. Following the R&D-based growth models tradition (Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), among others) the evolution of $N(t)$ can be modelled as

$$\dot{N}(t) = \delta u(t) N^{\phi}(t) - \eta N(t)$$

where $\delta > 0$ is a parameter which measures the rate at which R&D efforts generate an innovationb, $u(t)$ is the variable which measures the effort devoted to R&D (u measures the labor or capital devoted to research for instance), $0 < \phi \leq 1$ is the elasticity and $0 \leq \eta \leq 1$ is depreciation. Depreciation is the gradual decrease in the economic value of the innovations. Taking this into account, the following proposition presents the differential equation system that explains the evolution of the number of tourists at a destination where R&D efforts are taking place.

Proposition 2 *The number of tourists evolves according to the following differential equations system*

$$\dot{T}(t) = \alpha T(t)\left[1 - \frac{T(t)}{CN(t)}\right], \qquad (8)$$

$$\dot{N}(t) = \delta u(t) N^{\phi}(t) - \eta N(t) \qquad (9)$$

where C is a constant, $u(t)$ is the variable which measures the effort (share of labor or capital) devoted to R&D, $\alpha > 0$, $\delta > 0$, $0 < \phi \leq 1$, $0 \leq \eta \leq 1$ are parameters.

Proof. Let $s(t)$ be the solution of the following differential equation

$$\dot{s}(t)=\frac{\dot{N}(t)}{N(t)}\frac{1}{\alpha}\left[1+e^{\alpha(t-s(t))}\right] \quad s(0)=t_0. \tag{10}$$

From the fundamental theorem of existence and uniqueness of a solution of a differential equation, it is known that there exists a unique function $s(t)$ satisfying (10). By differentiating in (6), it is obtained that,

$$\dot{T}(t)=T(t)\left[\frac{\dot{N}(t)}{N(t)}-\frac{\alpha e^{-\alpha(t-s(t))}(1-\dot{s}(t))}{1+e^{-\alpha(t-s(t))}}\right].$$

Taking into account (10) the result is obtained.■

Note that variable $u(t)$ represents effort for R&D in tourism industry and it drives the evolution of diversity $N(t)$. This variable is exogenously given and will be determined by the entrepreneurship activity or government spending on infrastructures and services. If depreciation is nil and no investment is made, $u(t)=0$ for all t, then $\dot{N}(t)=0$, which would imply that the accumulated demand ceiling will remain constant, $\dot{K}(t)=0$. In this case we find the logistic pattern of growth proposed initially by Lundtorp and Wanhill (2001) (see Figure 9). In contrast, if $u(t)=\bar{u}>0$ for all t and $\phi=1$, a constant and steady effort is devoted to innovation in the tourism industry. In this case, variable $N(t)$ grows at a constant rate (exponential growth), as in Albaladejo and Martínez-García (2014) (see Figure 11). The following graphs simulate the evolution of the number of tourists $T(t)$ and diversity $N(t)$, equations (8) and (9), for different patterns of investment, $u(t)$, when depreciation is nil ($\eta=0$)

IMAGE 4 (Figures 9, 10 and 11)

Previous simulations assume that the choice of the decision rule $u(t)$ is made at time $t=0$ and the planner (government/entrepreneur) is committed to using this policy during the whole temporal horizon. In the real world, the planner observes the number of visitors $T(t)$ and the ceiling *K(t)=CN(t)* at each specific time t (estimates the congestion, $T/(CN)$, at each instant of time t) and then chooses an action according to the congestion level. As an example, the planner could chose $u=0$, that is, not to invest in tourism innovation, while the destination is not saturated (while $T/(CN)<1$), and $u=\bar{u}$, that is, a constant effort in tourism innovation while the destination is suffering congestion (while $T/(CN)\geq 1$). That is

$$u(t)=\begin{cases} 0 & \text{if} \quad T/(CN)<1 \\ \bar{u} & \text{if} \quad T/(CN)\geq 1 \end{cases}$$

which is called a feedback policy.

The resulting evolution of tourists $T(t)$ and activities $N(t)$ are depicted in the following graphs.

IMAGE 5 (Figure 12)

Note that the simulated evolution of the number of tourists in Figure 12 resembles the same behavior as the number of tourists in Bornholm depicted in Figure 7 of the previous section.

Nevertheless, although in the previous examples we have assumed that depreciation is nil, any investment on infrastructures suffer from depreciation. That is, there exits a gradual decrease in the economic value of the tourism infrastructures either through physical depreciation, obsolescence or changes in the demand for the services of the infrastructures in question. The following figure simulates the evolution of the number of tourists T(t) and diversity N(t), equations (8) and (9), if depreciation occurs.

IMAGE 6 (Figure 13)

Figure 13 shows the post-stagnation stage when the rate of investment cannot compensate for the depreciation of the tourism services and the decline is the result.

Concluding remarks

We have tested a multi-logistic growth model with data of visitors to Bornholm from 1912 to 2014. A fragment of these data allowed Lundtorp and Wanhill (2001, 2006) to propose the logistic growth model as a good mathematical approximation to the single life cycle of a tourist destination as put forward by Butler, but also to expose its limits where it only holds if there is one dominant market of repeat visitors. We have found that in the case of multiples markets a multilogistic model fits the entire series of data better. Taking into account this finding, we have proposed a new mathematical model based on the multilogistic growth pattern. This model represents the birth and the continuous superposition of new life cycles (new logistic growth patterns). Following the tradition of the R&D-based models of growth (Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992)), efforts to innovate are necessary for the maintenance of the process over time. The new multi-logistic growth model proposed in this paper resembles the six initial phases of the

TALC theory and the post-stagnation phase, which is suitable for mature destinations. Depending on the efforts of government and entrepreneurship, rejuvenation, stagnation or decline are possible outcomes.

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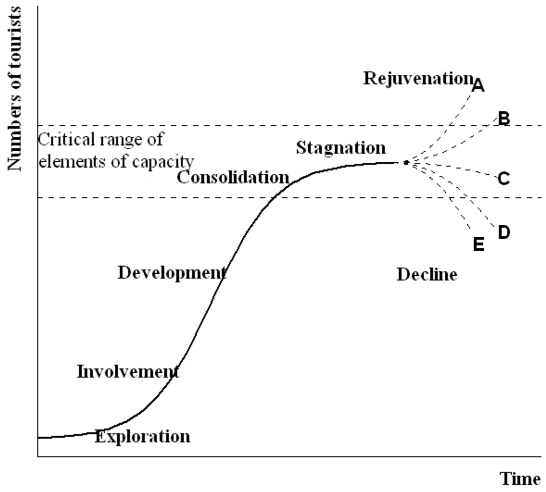

Figure 1. Evolution of tourist area according to the TALC. Source Butler (1980).

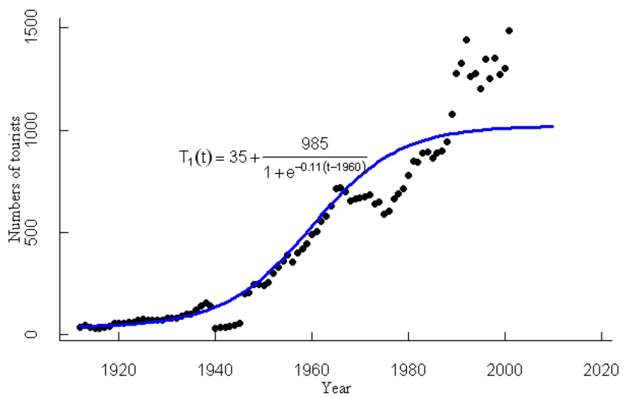

Figure 2. Passengers to Bornholm and the logistic curve. Lundtorp and Wanhill (2006).

Function with two phases of logistic growth:

$$T_2(t) = \frac{C_1}{1+e^{-(\alpha t-\beta_1)}} + \frac{C_2}{1+e^{-(\alpha t-\beta_2)}}$$

Param.	Estimate	Std. Error	t value
C_1	611.52812	28.40458	21.529
C_2	948.69419	40.01069	23.711
α	0.16316	0.01576	10.352
β_1	-6.37286	0.66879	-9.529
β_2	-12.49932	1.18588	-10.54

Table 1: Res. stand error: 77.86 on 92 d. of free.

Achieved convergence tolerance: 7.611e-07

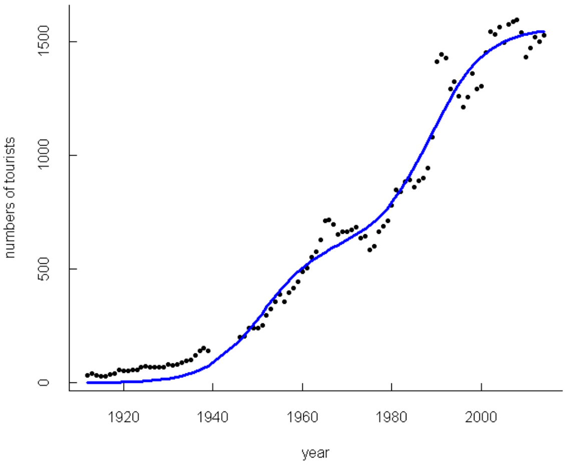

Figure 3

Function with three phases of logistic growth:

$$T_3(t) = \frac{C_1}{1+e^{-(\alpha t-\beta_1)}} + \frac{C_2}{1+e^{-(\alpha t-\beta_2)}} + \frac{C_3}{1+e^{-(\alpha t-\beta_3)}}$$

Param	Estimate	Std. Error	t value
C_1	114.28815	26.07883	4.382
C_2	538.68177	30.12384	17.882
C_3	860.34068	29.79452	28.876
α	0.22398	0.02092	10.709
β_1	-2.30125	1.01855	-2.259
β_2	-9.69816	0.98992	-9.797
β_3	-17.12245	1.58874	-10.777

Table 2: Res. stand error: 69.22 on 90 d. of free.

Achieved convergence tolerance: 7.054e-07

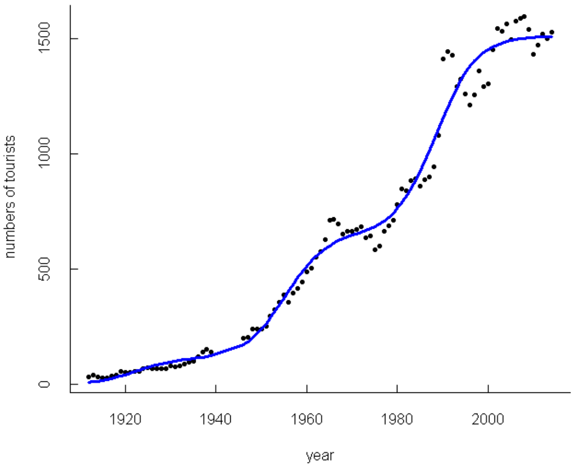

Figure 4

IMAGE 1

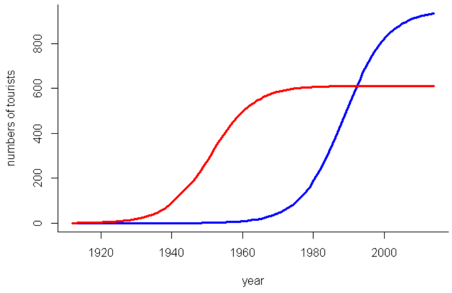

Figure 5.: Two logistic curves that define the function T_2.

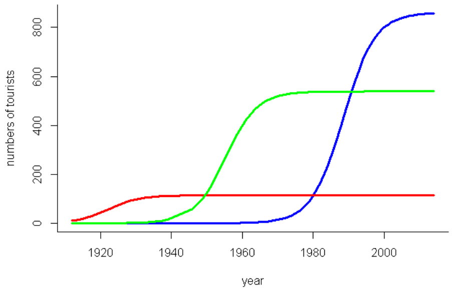

Figure 6.: Three logistic curves that define the function T_3.

IMAGEN 2

Function with four phases of logistic growth (T_4)

Param.	Estimate	Std. Error	t value
C_1	210.95229	28.43322	7.419
C_2	456.28035	31.32556	14.566
C_3	206.21827	52.13101	3.956
C_4	672.18822	55.58758	12.092
α	0.34783	0.04726	7.359
β_1	-7.39871	1.23378	-5.997
β_2	-15.95605	2.25352	-7.08
β_3	-31.07296	4.02277	-7.724
β_4	-25.91724	3.42026	-7.578

Table 3: Res. stand error: 67.33 on 88 d. of free.

Achieved convergence tolerance: 8.666e-08

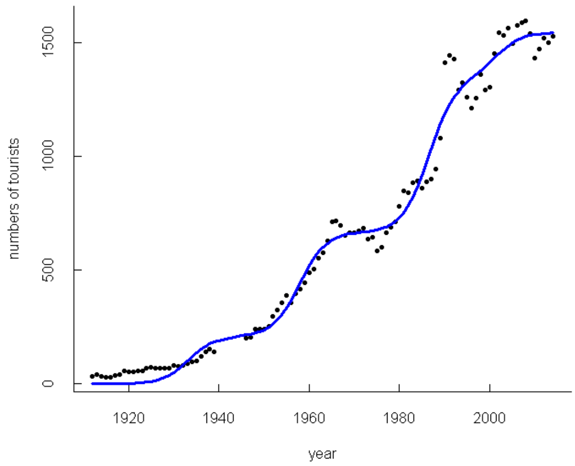

Figure 7

IMAGE 3

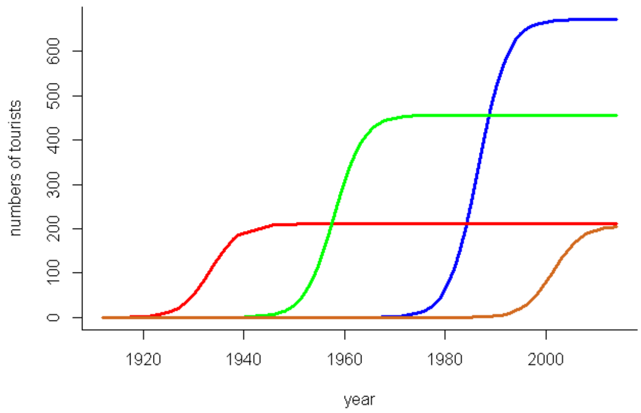

Figure 8.: Four logistic curves that define the function T_4.

	df	AIC	BIC
T_1	4	1153.007	1163.31
T_2	6	1127.005	1142.45
T_3	8	1106.043	1126.64
T_4	10	1102.491	1128.24

Table 4: Comparing models

No investment in R&D ($u(t) = 0$)[4]

$$\dot{T}(t) = 0.1T(t)\left[1 - \frac{T(t)}{CN(t)}\right],$$

$$\dot{N}(t) = 0$$

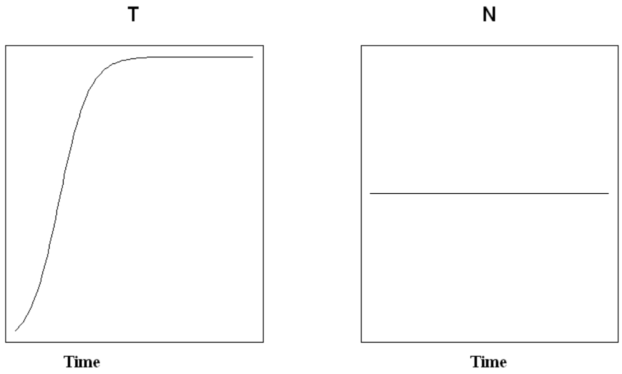

Figure 9

Positive investment in R&D

at a declining rate $\left(u(t) = \frac{1}{N(t)}\right)$

$$\dot{T}(t) = 0.1T(t)\left[1 - \frac{T(t)}{CN(t)}\right],$$

$$\dot{N}(t) = 0.009$$

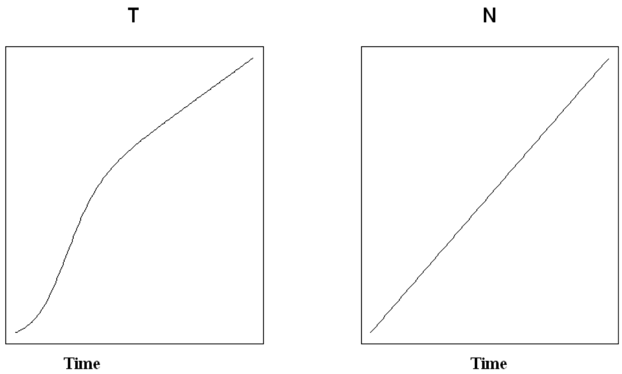

Figure 10

Positive investment in R&D

at a constant rate $(u(t) = 1)$

$$\dot{T}(t) = 0.1T(t)\left[1 - \frac{T(t)}{CN(t)}\right],$$

$$\dot{N}(t) = 0.009N(t)$$

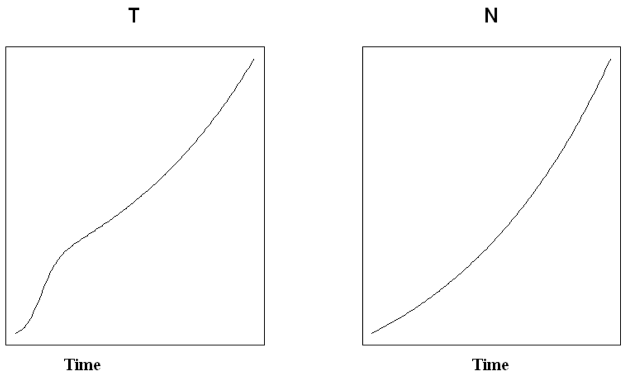

Figure 11

IMAGE 4

Feedback investment in R&D

$$\dot{T}(t) = 0.1T(t)\left[1 - \frac{T(t)}{CN(t)}\right],$$

$$\dot{N}(t) = 0.009\mathrm{u(t)N(t)}$$

$$\text{with } u(t) = \begin{cases} 0 & \text{if} T/N < 1 \\ 10^8 & \text{if} T/N \geq 1 \end{cases}$$

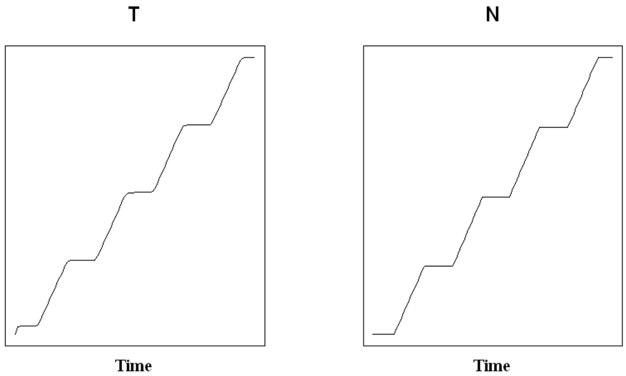

Figure 12

IMAGE 5

Positive investment in R&D at

a constant rate $(u(t) = 1)$

and depreciation $(\eta = 0.009)$

$$\dot{T}(t) = 0.1T(t)\left[1 - \frac{T(t)}{CN(t)}\right],$$

$$\dot{N}(t) = 0.009N^{1/2}(t) - 0.009N(t)$$

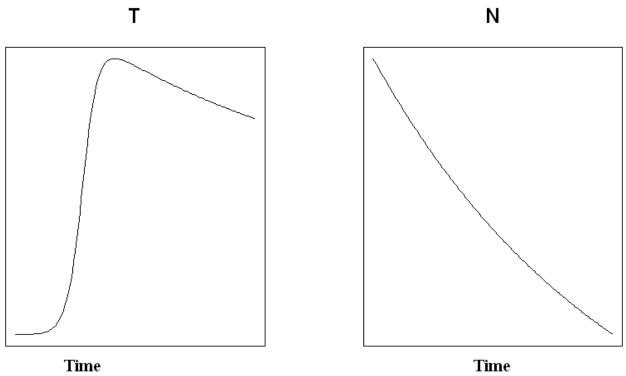

Figure 13

IMAGE 6

[1] A previous version of this paper used the available data up to 2001. Stephen Wanhill, with thanks to Carl Marcussenon from Bornholm, later provided us with the complete data series to 2014. The new data have allowed us to test our mathematical model with recent data and greatly enrich our study in this version. We are extremely grateful to both for theirgenerosity.

[2]In this paper we follow Lundtorp and Wanhill (2001) in considering T as the number of visitors to a destination. Other possibilities are allowed, as in Cole (2009), where the supply of rooms is the key variable.

[3]The data from the World War II period 1940-45 were excluded from the estimation because Bornholm was occupied by German, and then Soviet, troops until spring 1946.

[4]For simplicity, in these examples, we have identified the number of activities *N(t)* with the demand ceiling *K(t)*. That is, each activity has a ceiling C equal to 1.