

# Time, Quality and Growth

Francisco Alcalá

*(Universidad de Murcia)*

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Revised, June 2009

## Abstract

Consumption requires time. Also, higher-quality goods provide more utility per unit of time, though at a higher monetary cost. Since time is limited, higher income is decreasingly spent augmenting the quantity of consumption and increasingly spent upgrading its quality. After analyzing these consumer quantity/quality choices, the paper investigates its implications for growth. As a country develops, quality growth becomes increasingly important as a component of GDP growth. Furthermore, technical progress is increasingly quality-biased. Lower income inequality as well as progressive consumption taxes raise the scale of output while reducing average quality. This is positive for growth at early stages of development but may be negative at later stages. Results are broadly consistent with evidence on the composition of GDP growth, trade patterns of vertical specialization across countries, and the non-linearity of the impact of inequality on growth.

**Keywords:** Time Allocation, Product Quality, Growth, Inequality, Progressive Consumption Taxes. **JEL Classification:** O11, O15, O33.

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\*Universidad de Murcia and IVIE. First version of the paper: March 2008. This project was conducted while visiting the Economics Department at NYU whose hospitality is very much appreciated. I thank comments by participants at the 2009 Econometric Society North American Winter Meeting. Financial support from the Spanish Ministry of Science project ECO2008-02654/ECON and from Fundación Séneca de Murcia project 05710/PHCS/07 is gratefully acknowledged. Contact: falcala@um.es

# 1 Introduction

For most goods, production and expenditure can be seen as involving two dimensions: quantity and quality. For example, car companies may increase their output by increasing the number of cars they produce or by raising their quality, consumers may increase their expenditure in restaurants by dining out more often or by going to better restaurants, etc. Most macroeconomic models do not distinguish between these two dimensions. However, the distinction is increasingly important as suggested by the evidence presented below. This paper provides a new approach to the analysis of consumer quantity/quality choices and explores its implications for growth. In this second respect, the paper considers four issues: the share of quality growth as component of GDP growth; the quality bias of technical progress; the potential of progressive consumption taxes as a growth policy; and the effect of inequality on growth through the quantity/quality channel. These issues are analyzed within a very tractable model.

Although measuring quality is admittedly a difficult endeavor, the relevance of the quantity/quality distinction is becoming apparent in a number of areas, including the importance of quality growth as a component of GDP growth. Bils and Klenow (2001) estimate that annual quality growth averaged 3.7 percent for consumer durable goods in the US over 1980-1996 (about 60 percent of this growth would be wrongly accounted for as inflation by the Bureau of Labor Statistics, BLS). Moreover, Bils (2005) and (2009) estimates an annual quality growth for durables of about 5 percent since 1988. Additional calculations suggest that quality growth could account for a large portion of GDP growth. Moulton and Moses (1997) use data for 1995 and estimate that BLS methods may account for as much as one percent quality growth that year. To this one percent we must add unmeasured quality growth. The Boskin Commission Report (1996) argued that unmeasured quality change was the most important source of CPI upwards bias in the US and would be responsible for an approximate effect of 0.6 percent per year.<sup>1</sup> The importance of quality growth as a

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<sup>1</sup>The literature on the upward bias in the measurement of CPI inflation due to unmeasured quality growth is extensive. For example, Gordon (2006) estimates that, even after the thorough methodological revisions recently put in place by the BLS, there is still an upward bias in the measurement of CPI annual inflation of at least one percentage point, which is due to a large extent to unmeasured quality upgrading. See also Shapiro and Wilcox (1996) and Gordon (2005); and Bils and Klenow (2001) and Bils (2005) for further references.

component of GDP growth seem to have increased over time since backwards extrapolation of recent estimations of CPI bias appears to be unrealistic.<sup>2</sup>

Recent international trade literature also provides evidence of the increasing importance of the quantity/quality distinction in describing current patterns of international trade. On the demand side, trade is characterized by richer countries importing relatively more from countries that produce higher-quality goods (Hallak 2006). Along the same line, country pairs with similar income distributions have similar import price distributions, suggesting similar quality distributions of imports (Choi, Hummels and Xiang 2008). On the supply side, horizontal country specialization across goods is losing importance relative to vertical (quality) specialization within goods. Both low- and high-wage countries are increasingly exporting the same kinds of goods though of different average qualities. Richer countries tend to export the higher-quality varieties (Schott 2004; Hummels and Klenow 2005).

This paper provides a model that is consistent with these facts. The starting point is the analysis for consumer quantity/quality choices, which can be outlined as follows. Consumption requires time. Individuals need time to listen to a concert, travel for pleasure, or play with a video game. Since time is limited, this reduces the possibility of increasing utility by increasing the quantity of consumption (i.e., by increasing the number of units of the goods being consumed). In turn, higher-quality goods provide higher utility per unit of time allocated to consumption though at a higher monetary cost. The consequence of this is that higher income is decreasingly spent augmenting the quantity of consumption and increasingly spent upgrading its quality.<sup>3,4</sup> Most models considering the quality dimension of

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<sup>2</sup>According to Gordon (2005): "While the CPI may have overstated inflation in the mid-1990s by about one percent per year, as concluded by the Boskin Commission, it does not make sense to extrapolate that rate of bias backwards over long periods of time. The "Hulten-Bruegel paradox" shows that any such exercise in backward extrapolation yields levels of real consumption two or four centuries ago that are implausibly low, barely providing an average household with a pound of potatoes per day, with nothing left over for clothing or shelter."

<sup>3</sup>In other words, obtaining utility through consumption has both a time and a monetary cost. Higher-quality goods provide utility at higher monetary cost but cheaper time cost. Thus, richer individuals consume higher quality.

<sup>4</sup>Some durable goods such as washing machines and vacuum cleaners save time for individuals. However, these goods are better seen as capital goods used for household production instead of as consumption goods (people enjoy the cleanliness produced by a vacuum but do not tend to enjoy spending time vacuuming the house). See for example the theoretical and quantitative analysis in Greenwood, Seshadri and Yorukoglu

goods typically assume a positive relationship between income and the demand for quality. However, this positive relationship is taken as a fact without exploring the underlying reasons. This paper shows that the positive relationship between income and the demand for quality can be explained as a consequence of the time constraint and the complementarity between consumption and time.<sup>5</sup>

The analysis of consumer quantity/quality decisions along the lines just described is embedded later in the paper into a growth model. The natural implication is that, as a country develops, raising the quality of output becomes increasingly important as a component of GDP growth, relative to quantity growth. This is consistent with the empirical evidence of the large importance of quality growth in recent times cited above and with the unlikelihood that these estimates could be extrapolated backwards.

Labor efficiency in the model has two dimensions, each linked to one of the two dimensions of production: quantity and quality. Endogenous growth is introduced by assuming learning-by-doing in each of these two dimensions: labor learns to work *faster and better*. The model predicts that technical progress is increasingly biased in favor of reducing the costs of producing higher-quality goods. Since richer countries use more advanced technologies, this quality-biased technical progress prediction is consistent with the cited international trade evidence showing that richer countries have a comparative advantage in producing higher-quality goods. It is also shown that the model can be calibrated to produce reasonable long run paths of GDP growth. Nonetheless, the simple endogenous growth model built in this paper seems to strongly overestimate the quality component of GDP growth. Thus, an important task for future research is to build and calibrate more flexible models that can do a better job at matching the quantity and quality components of GDP growth, as more thorough estimates of these components become available.

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(2005). The concept of household production was introduced by Becker (1965). The allocation of time to different types of consumption, leisure, and home production have been investigated by Mark Aguir and Erik Hurst in a series of papers (see, for example, Aguir and Hurst 2007 and 2008).

<sup>5</sup>The literature has paid little attention to the implications of the complementarity between consumption and time in spite of the early analysis by Becker (1965). Outstanding recent exceptions are Goolsbee and Klenow (2006), and Hall and Jones (2007). The underlying key argument in this second paper is specially close to the one in the present paper: since individuals need time to obtain utility from consuming more goods, they spend a larger share of income on extending life expectancy as they get richer. Still, the connection between the time constraint and the demand for higher-quality goods, which is the starting point in this paper, seems to have been completely neglected in the literature.

The model also has intuitively appealing implications for policy and for the growth impact of income inequality. The model predicts that the quantity/quality composition of output affects the intensity and quality bias of technical progress and, therefore, affects growth. The quantity/quality composition of output is determined by consumer choices which in turn can be influenced by progressive consumption taxes. Therefore, progressive consumption taxes can be used as a growth policy. In the model, learning in both the quantity and the quality dimensions of efficiency go to zero as the scale of production goes to zero. It is then shown that growth can be enhanced at early stages of development (when efficiency and production are small) by shifting the quantity/quality composition of GDP in favor of a larger scale. Therefore, charging higher taxes on higher-quality goods heightens growth at early stages. Thus, the model provides a new argument for taxing luxury goods in developing countries. However, the positive impact of progressive consumption taxes can change to negative at later stages of development. At later stages, output has already reached a large scale whereas high efficiency at producing high quality becomes more valuable as consumers get richer.

Inequality also affects growth by means of a similar mechanism. Since preferences are non-homothetic with respect to quantity/quality pairs, income distribution affects the composition of output. In turn, the composition of output affects the intensity and quality bias of technical progress. This mechanism is the last issue explored in the paper. The model predicts that higher inequality hinders growth at early stages of development. The reason is that higher inequality raises average quality of GDP but reduces the volume of output. And, as already noted, reaching a large scale of output is more effective to foster technical progress at early stages of development, than producing high quality. Nonetheless, again, the sign of the inequality-growth relationship can change at more advanced stages of development as quality-biased technical progress becomes more valuable.

The actual impact of inequality on growth is still a debated empirical issue. However, the predictions here are consistent with most of the recent empirical evidence, as discussed in the corresponding section below. Especially relevant in connection with this paper is the evidence in Barro (2000) and (2008) who controls for most mechanisms previously suggested in the literature and still finds a significant non-linear impact of inequality and growth. Specifically, he finds a negative impact of inequality on growth among the group of less developed countries, which vanishes or even turns into a positive impact among the group of richer countries, as predicted by this model. There is also some scattered historical evidence

supporting a favorable contribution of lower income inequality to early industrialization, although its quantitative importance is unclear.<sup>6</sup>

The paper is organized as follows. In the next section I review some additional related literature on growth. The partial equilibrium analysis of the consumer's quantity/quality decisions is carried out in Section 3. Section 4 embeds this analysis into an exogenous technical-change growth model. It explores the quantity/quality composition of GDP growth along different stages of development. Technical change is endogenized in Section 5 assuming a two-dimensional learning-by-doing process in a model where technical progress can be quality biased. Section 6 explores the potential role of progressive consumption taxes as a growth policy. Section 7 introduces agent heterogeneity to study how income inequality affects growth through the quantity/quality composition mechanism. Section 8 summarizes and concludes.

## 2 Related Growth Literature

This paper can be framed within the related growth literature as follows. Economies expand their per capita consumption as they get richer by consuming more units of each good (the intensive margin), a wider set of goods (the extensive margin), and higher average quality (the quality margin). Most of the growth literature focuses on the intensive margin. Grossman and Helpman (1991) and Greenwood and Uysal (2005), among others, have analyzed the extensive margin from different perspectives. This paper focuses on the quality margin in

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<sup>6</sup>More unequal economies would spend a larger share of income in artisan production of luxury goods consumed by the elite, which has a limited scope for productivity gains. In contrast, more equal distributions of income would be more favorable to the development of industrial mass production of lower-quality goods. For example, the United States developed a larger spectrum of mass-production industries than England (where production was often oriented towards higher quality) in the first half of the nineteenth century. This seems to have been the consequence of a large demand by a wide number of middle class farmers and was at the root of its economic superiority in the twentieth century (Rosenberg 1972). The data by Milanovic, Lindert and Williamson (2008) are also consistent with the first industrial countries having relatively low *inequality extraction ratios* at the time of industrialization (the inequality extraction ratio is a measure of actual inequality with respect to the maximum feasible inequality given the economy's resources). The model in this paper captures in a simple way the positive relationship at early stages of development between more equality, mass production, and technical progress at producing low quality goods; as opposed to small scale artisan production of high quality goods and low technical progress.

conjunction with the intensive margin.

There is some important work related to the quality margin. Grossman and Helpman (1991) analyze quality-improving innovations across the existing set of consumption goods. However, different qualities are assumed to be perfect substitutes. Hence all individuals consume the cheapest quality-adjusted good and there is no room for a relationship between income and the demand for quality. The closest growth model with quality differentiated consumption goods is Stokey (1988). In Stokey (1988), as in this paper, consumption expands in both the quantity and the quality dimensions, and a positive relationship between income and the quality of consumption is obtained. Nonetheless, both the formulation and the applications here are different. A primary difference is that preferences in Stokey (1988) do not have enough structure to generate a specific pattern of the quantity/quality composition of growth. That is, output quality is increasing over time, but the importance of quality growth as a component of GDP growth could be decreasing. The time-to-consume constraint considered here introduces such a structure. There is also an emphasis in this paper on building a very tractable model that is easily amenable to quantitative analysis.

Demand non-homotheticities create a link between income distribution and the composition of output, which in turn may affect efficiency, technical progress, and growth. There is a string of literature exploring these links to which the analysis in Section 7 of this paper is also related. Murphy, Shleifer and Vishny (1989) is a static model where lower inequality may help overcome the indivisibilities involved in modern industrial technologies. Zweimüller (2000), Matsuyama (2002), and Foellmi and Zweimüller (2006) are closer to this paper. They build dynamic models where income distribution affects efficiency and technical progress. However, their setting is very different from the one here. All these papers assume hierarchical preferences across a growing set of goods. Each individual consumes one or zero units of each good, and richer individuals consume all the goods that are consumed by poorer individuals, plus some additional goods. Hence these papers focus on the extensive margin of consumption. The model in this paper is instead set in a simpler quantity/quality space of preferences and a two-dimensional technology space. These differences bring about a distinctive mechanism and specific results about the interaction between inequality and growth.<sup>7</sup>

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<sup>7</sup>Two other related papers are the following. Mani (2001) investigates how the interaction between inequality and output composition affects human capital accumulation. Zweimüller and Brunner (2005)



### 3 Time to Consume and the Preference for Quality

This section analyzes consumer quantity/quality choices assuming that consumption requires time. It develops the basic argument in a simple form that can be useful for growth analysis in the remaining sections. In order to clarify the assumptions, it is convenient to first consider an economy where there is only a single quality variety of the good.

#### 3.1 One Good and a Single Quality

There is a single good which is produced in a single quality variety. Utility depends on the number of units  $x$  being consumed and the time allocated to their consumption. Allocating more time to consuming a given unit of the good increases the utility it provides. It is assumed that the individual allocates consumption time evenly across all the units being consumed. The amount of time per unit of consumption is denoted by  $\omega$ .<sup>8</sup> Total time allocated to consumption is assumed to be exogenous and normalized to be 1.<sup>9</sup> Consumer utility is given by the following  $C^2$  function:

$$U = U(x, \omega); \tag{1}$$
$$\lim_{x \rightarrow 0, \omega \rightarrow \infty} U = 0, \quad U_x \geq 0, \quad U_\omega \geq 0;$$

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consider an economy with a quality-differentiated good and a non-differentiated good in order to analyze how income distribution affects the intensity of innovative activities in the quality-differentiated good. Depending on the equilibrium regime, the three classes of consumers (poor, middle class, and rich) may or may not consume the same quality of the differentiated good; and lower inequality may have a positive effect of innovation. See Foellmi and Zweimüller (2006) for further references on related literature.

<sup>8</sup>This assumption could be derived from a framework where the consumer chooses how much time to allocate to the consumption of each unit of the good, and where the marginal utility of the time allocated to each unit is decreasing.

<sup>9</sup>The complementarity between consumption and time has important implications for the allocation of time between work and leisure (or consumption). For example, if consumption and time are perfect complements and there is no quality differentiation, higher wage necessarily implies lower labor supply. However, the main points in this paper can be made without considering individuals' labor supply decisions. Hence I simplify on this point.

where subscripts on  $U$  indicate partial derivatives. Clearly,  $\lim_{x \rightarrow 0, \omega \rightarrow \infty} U = 0$  above is just a normalization. Consumer maximizes (1) subject to income and time-to-consume constraints:

$$x \leq y, \quad (2a)$$

$$1 = x \cdot \omega, \quad (2b)$$

$$x \geq 0, \quad \omega \geq 0; \quad (2c)$$

where the price of the good is normalized to be 1 and  $y$  denotes income.

The analysis will require some additional assumptions. As in Hall and Jones (2007), it is convenient to set them in terms of the elasticities of utility, which in this model are  $\eta_x(x, \omega) \equiv \frac{x}{U} U_x$  and  $\eta_\omega(x, \omega) \equiv \frac{\omega}{U} U_\omega$ . I will consider the following:

**Assumption 1**  $\lim_{x \rightarrow 0, \omega \rightarrow \infty} \eta_x(x, \omega) > 0$  and  $\lim_{x \rightarrow 0, \omega \rightarrow \infty} \eta_\omega(x, \omega) = 0$ . Also,  $\frac{\partial \eta_x(x, \omega)}{\partial x} \leq 0$ ,  $\frac{\partial \eta_\omega(x, \omega)}{\partial \omega} \leq 0$ ,  $\frac{\partial \eta_\omega(x, \omega)}{\partial x} \geq 0$ , and  $\frac{\partial \eta_x(x, \omega)}{\partial \omega} \geq 0$ , with at least one strict inequality.

The elements of this assumption seem self-explanatory. The elasticities of utility with respect to consumption and time are decreasing, whereas  $\frac{\partial \eta_\omega(x, \omega)}{\partial x} \geq 0$  and  $\frac{\partial \eta_x(x, \omega)}{\partial \omega} \geq 0$  convey the complementarity hypothesis of time and consumption. Note that as a result of the time constraint, the amount of time available to consuming each unit of the good decreases by  $1/x^2$  when the consumer increases the quantity of consumption. Taking this into account, we have  $\frac{dU}{dx} = \frac{U}{x} [\eta_x(x, \omega) - \eta_\omega(x, \omega)]$ . Hence, Assumption 1 implies that  $\lim_{x \rightarrow 0, \omega \rightarrow \infty} \frac{dU(x, \omega)}{dx} > 0$  and that  $\frac{d^2U(x, \omega)}{dx^2} < 0$ .<sup>10</sup>

If consumption requires time, it also seems reasonable to assume that there is a limit to the number of goods that individuals can enjoy per unit of time.<sup>11</sup> If time and consumption are perfect complements (for example, watching movies), individuals may not have the time to consume more units even if they would like to do so should they have more time. Or, more generally, if time and consumption are only imperfect complements, total utility may decrease if a limited amount of time is allocated to the consumption of too many units

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<sup>10</sup>A alternative formulation of sufficient conditions for  $d^2U(x, \omega)/dx^2 < 0$  is  $\rho_x \equiv -xU_{xx}/U_x \geq 1$ ,  $\rho_\omega \equiv -\omega U_{\omega\omega}/U_\omega \geq 1$ , and  $U_{x\omega} \geq 0$ , with at least one strict inequality.

<sup>11</sup>It may be argued that consumers can enjoy different goods at the same time. For example, one can simultaneously enjoy wearing nice shoes, listening to music, driving a comfortable car, etc. However, at every point in time, the existing number of different types of goods is finite. One can then make the argument taking as the unit of consumption the whole finite bundle of goods that can be consumed simultaneously.

of the good.<sup>12</sup> In other words, for a quantity of consumption  $x$  sufficiently large, the time allocated to consuming each unit would be so small that the overall marginal utility of further increasing the quantity of consumption (and therefore, further reducing the time to consume each unit) would be zero or negative. I will consider the following alternative assumptions.

**Assumption 2a**  $\lim_{x \rightarrow \infty, \omega \rightarrow 0} \eta_x(x, \omega) = 0$  and  $\lim_{x \rightarrow \infty, \omega \rightarrow 0} \eta_\omega(x, \omega) > 0$ .

**Assumption 2b**  $\lim_{x \rightarrow \infty, \omega \rightarrow 0} \eta_x(x, \omega) < \infty$  and  $\lim_{x \rightarrow \infty, \omega \rightarrow 0} \eta_\omega(x, \omega) = \infty$ .

It is equivalent for the results that follow to assume either Assumption 2a or 2b. The important implication of any of the two assumptions is that  $\lim_{x \rightarrow \infty, \omega \rightarrow 0} [\eta_x(x, \omega) - \eta_\omega(x, \omega)] < 0$ . Hence, by continuity, there exists  $\bar{x}$ ,  $0 < \bar{x} < \infty$ , such that  $\eta_x(\bar{x}, \frac{1}{\bar{x}}) - \eta_\omega(\bar{x}, \frac{1}{\bar{x}}) = 0$ . Therefore, under Assumption 2a or 2b, utility maximization implies  $\lim_{y \rightarrow \infty} x^*(y) = \bar{x}$ . Thus, as a result of the complementarity between consumption and time, individuals may seem satiated (in the sense that they stop consuming more units even if their income is not binding) even if preferences do not display any satiation point; i.e., even if  $U_x(x, \omega) > 0$ , for any  $x > 0$  and  $\omega > 0$ .

Recall that the arguments in this subsection are made for an economy where individuals do not have the possibility of increasing the quality of the goods they consume. In the next subsection, it is shown that consumers' response to the time-to-consume constraint is to spend an increasing share of their income upgrading the quality of consumption instead of increasing its quantity.

### 3.2 A Continuum of Qualities

Let us now introduce quality. There is a single good which can be consumed along a continuum of quality varieties  $q \in [0, \infty)$ . Utility depends on the number of units  $x$  being consumed, their quality  $q$ , and per-unit of consumption time  $\omega$ . Higher-quality varieties provide more utility than lower-quality varieties when allocating the same amount of time

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<sup>12</sup>For example, individuals may not feel satiated by the number of books they read or the cities they visit. However, they may prefer taking more time to read one book and visit one city rather than rushing to read more books and visit more cities.

to their consumption. I simplify by considering a separable utility function of the form:

$$W(x, \omega, q) = U(x, \omega) \cdot V(q); \quad (3)$$

$$V(0) \geq 0, \quad V_q > 0, \quad V_{qq} \leq 0;$$

where  $U(x, \omega)$  is the same function as in the previous subsection. As before, the time allocated to consumption is taken as exogenous and normalized to be 1. Individuals maximize (3) subject to

$$y \leq x \cdot p(q), \quad (4a)$$

$$1 = x \cdot \omega; \quad (4b)$$

$$x \geq 0, \quad \omega \geq 0; \quad (4c)$$

where  $p(q)$  is price as a function of quality. Note that quality has no natural units of measurement. It is only a preference order over the different varieties of the same good, such that if two different varieties have quality indexes  $q'$  and  $q''$ , then  $q' > q''$  means that one unit of variety  $q'$  is always preferred to one unit of  $q''$ . Thus, quality indexes may be rescaled using an increasing function to obtain a simple specific pattern of prices as a function of qualities. This facilitates the analysis. Specifically, by an appropriate relabelling of qualities and without loss of generality, we can assume the following pattern of prices:<sup>13</sup>

$$p(q) = e^{q/\gamma}; \quad \gamma > 0. \quad (5)$$

The function  $V(q)$  is assumed to be defined after this relabelling. Since marginal utility of quality is assumed to be always positive, the budget constraint is always satisfied with equality. Taking (5) into account, utility maximization yields the following first order condition:

$$\eta_x(x, \omega) - \eta_\omega(x, \omega) = \gamma \frac{V_q(q)}{V(q)}. \quad (6)$$

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<sup>13</sup>To see that this, consider the original set of quality varieties to be  $\{Q\} = [0, \infty)$ . Quality varieties are assumed to be indexed (labelled) according to an increasing preference order such that if  $q' > q''$ , then one additional unit of variety  $q'$  is always preferred to one additional unit of  $q''$ . Let  $P(Q)$  be the price of quality  $Q$ .  $P(Q) : [0, \infty) \rightarrow [1, \infty)$  is assumed to be continuous, increasing, differentiable, and satisfying  $P(0) = 1$ . Relabel varieties as  $q \in [0, \infty)$  using the one-to-one increasing mapping  $\phi(Q) = q$  defined as  $\phi(Q) \equiv \gamma \ln P(Q)$ . Hence,  $Q = P^{-1}(e^{q/\gamma})$ . Therefore, the price of each relabelled quality is given by  $p(q) \equiv P(P^{-1}(e^{q/\gamma})) = e^{q/\gamma}$ . It may be noted that the parameter  $\gamma$  does not play any role in this section, so that we could just take  $\gamma = 1$ . However, it will play an important role in the endogenous growth model below so that it seems convenient to introduce it from the very beginning.

Substituting with (4b) into (6) implicitly defines a consumer-optimal mapping between  $x$  and  $q$ ,  $\psi(x) = q$ . This mapping is a continuous function  $\psi(x) : (0, b) \rightarrow (0, \infty)$ , such that  $b = \bar{x}$  if either Assumption 2a or 2b holds or  $b = \infty$  otherwise. Differentiating (6) with respect to  $x$  and  $q$  (after using (4b) to substitute for  $\omega$ ), and using Assumption 1 yields:

$$\begin{aligned} \frac{dq}{dx} &\equiv \psi' & (7) \\ &= \left[ -\frac{\partial \eta_x(x, \omega)}{\partial x} + \frac{\partial \eta_x(x, \omega)}{\partial \omega} \frac{1}{x^2} - \frac{\partial \eta_\omega(x, \omega)}{\partial \omega} \frac{1}{x^2} + \frac{\partial \eta_\omega(x, \omega)}{\partial x} \right] \frac{1}{\gamma} \frac{V(q)}{V_q(q)^2/V(q) - V_{qq}(q)} > 0. \end{aligned}$$

The mapping  $q = \psi(x)$  is depicted in Figure 1. In turn, given the individual's income  $y > 0$ , the frontier of consumer's attainable pairs  $(x, q)$  is obtained by substituting with (5) into the budget constraint (4a) with equality. This yields  $q = \ln(y/x)$ . The intersection of  $q = \psi(x)$  and  $q = \ln(y/x)$  determines the consumer's optimal pair  $(x^*, q^*)$  (see Figure 1). Then, (4b) determines  $\omega^*$ . Clearly, for any  $y > 0$  the two schedules cross in the positive quadrant. Moreover, higher  $y$  implies higher  $x^*$  and  $q^*$ . Hence, we have the following:

**Proposition 1** *Let Assumption 1 hold. As income rises, the quality of consumption increases.*

As argued in the Introduction, quality growth seems to be an increasingly important component of GDP growth. The result in Proposition 1 is not enough to guarantee this. It is also necessary that, as income rises, an increasing share of every income rise is spent upgrading the quality of consumption. Assumption 2a or 2b imply this pattern. Using  $q = \psi(x)$  to substitute in the income constraint  $y = xe^{q/\gamma}$  and differentiating yields that the share of an income rise  $dy$  that is spent increasing the quantity of consumption is  $p(q) \frac{dx}{dy} = \frac{1}{1+x \psi'/\gamma}$ . Reciprocally, the share of an income rise that is spent upgrading the quality of consumption is

$$x \frac{\partial p}{\partial q} \frac{dq}{dy} = \frac{x \psi'/\gamma}{1+x \psi'/\gamma}. \quad (8)$$

The following proposition conveys the message that as income rises, the share of every income rise that is spent upgrading the quality of consumption is increasingly larger.

**Proposition 2** *Let Assumption 1 and Assumption 2a or 2b hold. For any  $s$ ,  $0 < s < 1$ , there is an income level  $y(s)$  sufficiently large such that for  $y > y(s)$ , a share larger than  $s$  of every income rise is spent upgrading the quality of consumption.*

**Proof.** To check this proposition, recall that Assumption 2a or 2b imply that there is an upper bound  $\bar{x}$  to the quantity of consumption. Then, using the budget constraint  $y \leq x e^q$  and the assumption that marginal utility of quality is always positive, we have  $\lim_{y \rightarrow \infty} q = \infty$ . Consider now the two main factors in expression (7) for  $\psi'$ . Since  $\lim_{y \rightarrow \infty} x^*(y) \leq \bar{x}$ , Assumption 1 implies  $\lim_{y \rightarrow \infty} \left[ -\frac{\partial \eta_x(x, \omega)}{\partial x} + \frac{\partial \eta_x(x, \omega)}{\partial \omega} \frac{1}{x^2} - \frac{\partial \eta_\omega(x, \omega)}{\partial \omega} \frac{1}{x^2} + \frac{\partial \eta_\omega(x, \omega)}{\partial x} \right] > 0$ . In turn, either  $\lim_{q \rightarrow \infty} V(q) = \infty$ , or  $\lim_{q \rightarrow \infty} V_q(q) = 0$  and  $\lim_{q \rightarrow \infty} V_{qq}(q) = 0$ . In either case we have  $\lim_{q \rightarrow \infty} \frac{V(q)}{V_q(q)^2 / V(q) - V_{qq}(q)} = \infty$ . Hence, from (7) we have  $\lim_{y \rightarrow \infty} \psi' = \infty$ , so that  $\lim_{y \rightarrow \infty} x \frac{\partial p}{\partial q} \frac{dq}{dy} = \frac{x \psi' / \gamma}{1 + x \psi' / \gamma} = 1$ . Therefore, since (7) is continuous, for any  $s$ ,  $0 < s < 1$ , there is an income level  $y(s)$  sufficiently large such that  $y > y(s)$  implies  $x \frac{\partial p}{\partial q} \frac{dq}{dy} > s$ . ■

## 4 The Quantity/Quality Composition of Growth

The partial equilibrium analysis of consumer decisions in the previous section has a straightforward implication for the dynamics of the quantity/quality composition of growth: quality growth is an increasingly important component of GDP growth. This section develops this implication and also sets the framework for the following sections.

### 4.1 Technology

There is a single representative agent and a single good that can be produced along a continuum of qualities  $q \in [0, \infty)$ . Labor is the only factor of production and there are constant returns to scale producing any quality. Producing higher quality requires more labor per unit of output. Output at time  $t$ ,  $x_t$ , when producing quality  $q_t$  is given by

$$x_t = \frac{A_t L}{F(q_t)}.$$

where  $L$  is the labor input,  $A_t$  is a general efficiency parameter that evolves over time, and  $F(q)$  is a continuous, differentiable, strictly increasing function  $F(q) : [0, \infty) \rightarrow [1, \infty)$  that satisfies  $F(0) = 1$ . Labor supply is assumed to be constant and is normalized  $L = 1$ . By an appropriate relabelling of qualities and without loss of generality, we can

assume  $F(q) = e^{q/\gamma}$ . Hence, the production function becomes:<sup>14</sup>

$$x_t = \frac{A_t}{e^{q_t/\gamma}}. \quad (9)$$

Denote the wage at  $t$  by  $w_t$ . Assuming perfectly competitive markets, prices are

$$p_t(q) = \frac{e^{q/\gamma}}{A_t} w_t. \quad (10)$$

In this section, technical progress is exogenous:

$$g(A_t) = \theta > 0. \quad (11)$$

where  $g(\cdot)$  indicates the growth rate of the variable in parenthesis.

## 4.2 The composition of GDP growth

Consider the same consumer problem (3)-(4) as in the previous section, where prices are now given by (10). As in the previous section, from utility maximization we can obtain a consumer-optimal continuous, positively valued, and increasing mapping  $q_t = \psi(x_t)$ . Given  $A_t$ , expression (9) determines the frontier of the feasible pairs  $(x_t, q_t)$ , whereas  $q_t = \psi(x_t)$  determines consumers' choice among those pairs. The intersection between (9) and  $q_t = \psi(x_t)$  determines the instantaneous equilibrium pair  $(x_t, q_t)$  at time  $t$ . See Figure 2. Clearly, for any  $A_t > 0$  there is a unique pair  $(x_t, q_t) > (0, 0)$  solving this system. Moreover, as  $A_t$  grows,  $x_t$  and  $q_t$  will increase.

Denote GDP at time  $t$  at current prices by  $y_t$ :

$$y_t \equiv x_t p_t(q_t).$$

Therefore, GDP growth at constant prices for each quality variety  $p_t(q)$  is given by  $g(y_t) = g(x_t) + \frac{\partial p_t(q)/\partial q}{p_t(q)} \dot{q}_t$ . Then, using  $(\partial p_t/\partial q)/p_t = 1/\gamma$  from (10) yields:

$$g(y_t) = g(x_t) + \frac{q_t}{\gamma} g(q_t). \quad (12)$$

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<sup>14</sup>To see this, consider the original set of quality varieties to be  $\{Q\} = [0, \infty)$ . Relabel quality varieties as  $q \in [0, \infty)$  using the one-to-one increasing mapping  $\varphi(Q) = q$  defined as  $\varphi(Q) \equiv \gamma \ln F(Q_t)$ . Hence,  $Q = F^{-1}(e^{q/\gamma})$ . Therefore, substituting in the production function yields  $x_t = A_t L_t / F(Q_t) = A_t L_t / e^{q_t/\gamma}$ . Note that (9) is not an unusual production function in models with quality differentiation (see, for example, the important paper on international trade with quality differentiation Flam and Helpman 1987).

This expression separates GDP growth into a quantity and a quality component. The quantity component of GDP growth is:

$$g(x_t) = \frac{1}{1 + x_t \psi' / \gamma} g(y_t). \quad (13)$$

Clearly, GDP growth is given by the exogenous growth of efficiency  $A_t$ ; i.e.,  $g(y_t) = g(A_t) = \theta$ . This combined with Proposition 2 brings about the following:

**Proposition 3** *Let Assumption 1 and Assumption 2a or 2b hold. For any  $s$ ,  $0 < s < 1$ , there is a level  $A(s)$  of development in terms of technical efficiency such that for  $A > A(s)$ , the share of quality growth in total GDP growth is larger than  $s$  (i.e.,  $\frac{q_t}{\gamma} g(q_t) > s \cdot g(y_t)$ ).*

**Proof.** Constant positive growth  $g(y_t) = g(A_t) = \theta$  implies  $\lim_{t \rightarrow \infty} y_t = \infty$ . Then, using the arguments in the proof of Proposition 2, we have  $\lim_{t \rightarrow \infty} x_t \psi' / \gamma = \infty$ . Therefore, (13) yields  $\lim_{t \rightarrow \infty} g(x_t) = 0$ . Reciprocally,  $\lim_{t \rightarrow \infty} \frac{q_t}{\gamma} g(q_t) = g(y_t)$ . Since all the variables are continuous as a function of time and as a function of  $A_t$ , the result in the proposition follows. ■

This result is consistent with the seemingly increasing importance of quality growth as a component of GDP growth that was suggested in the Introduction.

## 5 Endogenous Growth with Quality-Biased Technical Progress

As noted in the Introduction, recent international trade literature shows that richer countries tend to have a comparative advantage in producing higher quality goods. Since richer countries have more advanced technologies, a potential explanation for this fact is that technical progress tends to be quality biased. That is, as a country develops, efficiency producing the higher qualities increases faster than efficiency producing the lower qualities. This section builds a model where technical progress shows this bias.

The model introduces endogenous growth in the simple form of learning by doing. Repetition of tasks improves skills and knowledge that in turn help both the speed and quality of work (i.e., experience helps produce goods faster and better). Hence technical progress comes along two dimensions: (i) general efficiency in producing any quality variety (which is captured in the production function by the variable  $A_t$ ); and (ii) relative efficiency in producing higher qualities (which is captured in the production function by the variable  $\gamma_t$ ). It



is assumed that, for any given quantity of output, the higher is the quality being produced, the more quality-biased is the resulting learning.<sup>15</sup>

## 5.1 The Model

**Technology** Consider the same production function as before except that now both technological parameters  $A_t$  and  $\gamma_t$  are subject to progress due to learning-by-doing. Labor supply is constant and normalized to be 1. Thus, output at time  $t$  when producing quality  $q_t$  is given by:

$$x_t = \frac{A_t}{e^{q_t/\gamma_t}}; \quad (14)$$

As already noted,  $A_t$  is a general efficiency parameter, whereas  $\gamma_t$  governs the relative efficiency in producing higher-quality goods. Technical progress in general efficiency  $A_t$  evolves over time as a result of a standard learning by doing process, as in Krugman (1987) and Lucas (1988):

$$\dot{A}_t = \theta_1 x_t - \delta_1 A_t; \quad \theta_1 > \delta_1 > 0. \quad (15)$$

In turn,  $\gamma_t$  is linked to both the quantity and quality being produced. For example, accumulated experience in sewing shirts in a clothing factory may bring about not only a higher number of sewed shirts per unit of labor but also improvements in skills and techniques for carrying out more accurate seams. Or, in package delivery, accumulated experience may help develop techniques to reduce misplacements (which is a quality characteristic), besides increasing the number of deliveries per worker. In general, repeating a task helps improve not only the speed at which the task is performed but also the quality of the result. Moreover, the higher the output quality being targeted in a production process, the more likely it is that learning will be quality-biased. Still, as the number of units being produced tends to zero, learning would also tend to zero even if the quality being produced is very high. These qualitative circumstances are embedded in a simple way in the following law of motion for  $\gamma_t$ :

$$\dot{\gamma}_t = \theta_2 q_t x_t - \delta_2 \gamma_t; \quad \theta_2 > \delta_2 \geq 0. \quad (16)$$

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<sup>15</sup>Learning by doing may be considered as an initial simple formulation for endogenous technical progress. Still, a model with intentional R+D would likely bring about similar qualitative results on the increasing quality bias of technical progress. The reason is that incentives for quality-biased R+D (as opposed to quality neutral R+D) are likely to increase as consumers spend a larger share of any income rise on upgrading the quality of consumption.

Obsolescence parameters  $\delta_1$  and  $\delta_2$  may be justified in terms of a succession of finitely lived representative agents whose skills have to be replaced.<sup>16</sup> At any rate, results are obtained assuming that  $\delta_1$  and  $\delta_2$  are small. In fact, the same central results can be obtained with  $\delta_1 = \delta_2 = 0$ . However,  $\delta_1 > 0$  brings about the existence of a steady state with constant rates of growth, which seems interesting to consider.

**Utility** I now simplify by considering a particular case of the utility function in previous sections that delivers an explicit solution. The representative agent maximizes the following instantaneous utility function subject to the same constraints (4a)-(4b)-(4c) as before:

$$U_t = \frac{\omega_t}{e^{\sigma/x_t}} q_t. \quad (17)$$

As in the previous section, the production function (14) implies that perfect-competition prices are given by  $p_t(q) = w_t e^{q/\gamma_t} / A_t$ . This, together with the first order conditions of utility maximization, brings about the following optimal relationship between quantity and quality of consumption:

$$\frac{q_t}{\gamma_t} = \frac{x_t}{\sigma - x_t}. \quad (18)$$

## 5.2 Equilibrium and Growth

**Instantaneous Equilibrium** Define  $z_t \equiv q_t/\gamma_t$ . This change in variable will allow solving first a one-state variable ( $A_t$ ) system with two control variables ( $x_t$  and  $z_t$ ), instead of dealing with a two-state variable system with two control variables ( $A_t$  and  $\gamma_t$ ; and  $x_t$  and  $q_t$ , respectively). Expressions (14) and (18) are now rewritten as:

$$x_t = \frac{A_t}{e^{z_t}}; \quad (19)$$

$$z_t = \frac{x_t}{\sigma - x_t}. \quad (20)$$

Given  $A_t$ , expression (19) sets the feasible pairs  $(x_t, z_t)$  at time  $t$ , whereas (20) determines consumer choice among those pairs (see Figure 3). Note that  $x/(\sigma - x)$  is increasing in  $x > 0$  with  $\lim_{x \rightarrow 0} \frac{x}{\sigma - x} = 0$ , whereas  $\ln(A_t/x)$  is decreasing in  $x$ . Clearly, for any  $A_t > 0$  there is a unique pair  $(x_t, z_t) > (0, 0)$  solving (19)-(20). Both  $x_t$  and  $z_t$  are continuous and strictly increasing in  $A_t$ . Then, given  $\gamma_t$ ,  $q_t = z_t \gamma_t$  determines output quality  $q_t$ .

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<sup>16</sup>Knowledge, experience, and skills are embodied in individuals that are subject to a life cycle.

**GDP Growth** GDP at time  $t$  is  $y_t \equiv x_t p_t(q_t)$ . GDP growth at time  $t$  at constant prices  $p_t(q)$  is given by  $g(y_t) = g(x_t) + \frac{\partial p_t(q)/\partial q}{p_t(q)} \dot{q}_t$ . Since  $(\partial p_t/\partial q)/p_t = 1/\gamma_t$  and using also (14), we have

$$g(y_t) = g(x_t) + \frac{q_t}{\gamma_t} g(q_t) = g(A_t) + \frac{q_t}{\gamma_t} g(\gamma_t). \quad (21)$$

This expression provides two approaches to GDP growth. From the point of view of the composition of output, GDP grows in both the quantity and the quality dimensions. From the point of view of the source, GDP grows due to general as well as quality-biased technical progress. The importance for GDP growth of consumption quality upgrading  $g(q_t)$  and quality-biased efficiency improvements  $g(\gamma_t)$  depends on the elasticity of prices with respect to quality  $(\partial p_t/\partial q)q_t/p_t$ ,  $(\partial p_t/\partial q)q_t/p_t = q_t/\gamma_t \equiv z_t$ . Clearly, there is no reason to expect  $g(x_t)$  to be equal to  $g(A_t)$  or  $g(q_t)$  to be equal to  $g(\gamma_t)$  (in fact, this will only be the case in the steady state). The behavior of GDP growth from the point of view of the composition of output is the same as in the model in Section 4. Hence the remainder of this section focuses on the analysis of GDP growth from the point of view of technical progress characteristics.

From (19), (15), and (16) we have:<sup>17</sup>

$$g(A_t) = \frac{\theta_1}{e^{z_t}} - \delta_1. \quad (22)$$

$$g(\gamma_t) = \theta_2 z_t x_t - \delta_2. \quad (23)$$

**Steady State** This economy has a unique steady state with strictly positive growth. In this steady state, the volume of output is constant whereas quality grows at a constant positive rate. The following proposition characterizes this using asterisks to denote steady state values.

**Proposition 4** *For  $\delta_2$  sufficiently small, the model has a unique steady state with constant values  $A^* > 0$ ,  $x^* > 0$ ,  $z^* > 0$ ,  $g_q^* = g_\gamma^* > 0$ , and  $g_y^* = z^* g_\gamma^*$ . Moreover,  $g_y^*$  is increasing in  $\theta_1$  and  $\theta_2$ , and decreasing in  $\delta_1$  and  $\delta_2$ .*

**Proof.** Substituting with  $g_A^* = 0$  in (22) yields  $z^* \equiv (q/\gamma)^* = \ln(\theta_1/\delta_1) > 0$ . With this, expressions (19) and (20) yield  $x^* = \sigma \frac{\ln(\theta_1/\delta_1)}{1 + \ln(\theta_1/\delta_1)}$  and  $A^* = x^* \frac{\theta_1}{\delta_1}$ . Then,  $g(z_t) = g(q_t) - g(\gamma_t)$

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<sup>17</sup>Note from (23) that, given the parameters of the economy and initial technological conditions, non-negative growth rates of  $\gamma_t$  (and of GDP) require  $\delta_2$  to be sufficiently low.

implies  $g_q^* = g_\gamma^*$ , and (23) yields  $g_q^* = g_\gamma^* = \theta_2 z^* x^* - \delta_2$  (which is positive for  $\delta_2$  sufficiently small). Finally, (21) yields  $g_y^* = z^* g_\gamma^*$ . ■

In Figure 3, the  $z = \ln(A_t/x)$  schedule shifts over time towards the North-East until it crosses the  $z = x/(\sigma - x)$  schedule at point  $z^* = \ln(\theta_1/\delta_1)$ , which corresponds to the steady state.

**Transitional Dynamics** The system (19)-(20)-(22) can be solved independently from the rest of equations. Denote the initial condition by  $A_0$ . For  $A_0 < A^*$ , the schedule (19) at  $t = 0$  crosses (20) to the South-West of  $(x^*, z^*)$  in Figure 3. It is easy to see that the economy converges monotonically to its steady state.<sup>18</sup>

Now, using (21), we can characterize GDP growth from the point of view of the quality bias of technical progress at different stages of development.

**Proposition 5** *Technical progress is mostly quality-neutral at early stages of development (which are characterized by low levels of general efficiency  $A$ ), but becomes increasingly quality-biased over time.*

**Proof.** See Appendix ■

This evolution of technical progress occurs in parallel to the shift of demand towards higher-quality goods.

### 5.3 A Numerical Illustration

In spite of its simplicity, this framework may deliver reasonable long run paths for per capita GDP growth. This section provides a numerical exercise.<sup>19</sup> The Neoclassical model predicts decreasing growth rates, whereas standard endogenous growth models usually predict constant rates. However, the secular pattern of growth in advanced economies since the industrial revolution has been one of increasing growth rates that seem to have stabilized somewhat below 2-percent in the last half century. For example, estimates of per worker

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<sup>18</sup>Given  $0 < A_0 < A^*$ , (19)-(20) imply  $0 < x_0 < x^* < \sigma$ ,  $0 < z_0 < z^* = \ln(\theta_1/\delta_1)$ . Then,  $g(x_t) = g(A_t) \frac{\sigma - x_t}{2\sigma}$  (which can be obtained using (22), (19), and (20)) implies  $g(A_t) > 0$  and  $g(x_t) > 0$ . Then,  $g(z_t) = g(x_t) [1 + z_t]$  implies  $g(z_t) > 0$ . Therefore the system converges monotonically to  $(A^*, x^*, z^*)$ .

<sup>19</sup>Notwithstanding, the stripped-down simplicity of the technical progress functions assumed in the model prevents obtaining quantitatively plausible long run paths for the division of growth between quantity and quality growth.

output growth in England at the beginning of the ninetieth century go from 0.35–percent to 1.3–percent (see Feinstein 1981, and Crafts and Harley 1992). On the other hand, US per capita growth in the recent decades has averaged about 1.7–percent (though, as discussed in the Introduction, this number may underestimate actual growth due to unmeasured quality growth). In this subsection the model is discretized and calibrated so that it roughly matches the long run per capita growth path of the most advanced economies in the last two centuries along those numbers. The calibration generates an initial per capita GDP growth of 0.72–percent (which is roughly consistent with the mean of the cited estimates for England at the beginning of the XIXth century) and yields a steady state growth of 1.8–percent (which is consistent with the overall observed trend in the last decades).

Parameter values are set as follows. Since skills and knowledge are embodied in individuals, annual depreciation rates  $\delta_1 = \delta_2 = 0.025$  seem reasonable. Furthermore, I set  $\sigma = 3$ ,  $\theta_1 = 0.054$ ,  $\theta_2 = 0.048$ . Finally, the initial value  $A_0$  is chosen such that  $x_0 = 1$ . Figure 4 draws the annual rates of growth of GDP for 300 years. As noted, calibrated GDP growth starts at an annual rate of 0.72–percent and then shows a rapid acceleration, doubling after 120 years and surpassing 1.68–percent after 150 years.

GDP growth is also divided in Figure 4 into the quantity growth component,  $g(x_t)$ , and the quality growth component,  $z_t g(x_t)$ . Consistent with Proposition 4, quantitative growth is the main source of growth at initial stages. Then, it is gradually substituted by quality growth. Although an empirical assessment of the secular paths of these two components of growth does not yet exist, their paths in Figure 4 seems implausible (the calibration seems to overrate quality growth in late periods). Future work may find it necessary to introduce more flexible functional forms for technical progress in order to generate more reasonable secular paths of these two components, as estimates of these paths become available. Introducing population growth will also enhance the importance of quantity growth.<sup>20</sup>

## 6 Growth Policy: Progressive Consumption Taxes

Progressive consumption taxation (i.e., taxing higher-quality or *luxury* goods at higher rates) has been discussed from different perspectives such as the impact on aggregate savings and

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<sup>20</sup>Note that population growth would be directly added to quantity GDP growth and would likely reduce the quality bias of technical progress.

the distribution of wealth.<sup>21</sup> This section discusses its possible role as a growth policy through its influence on the composition of demand. Note that different quantity/quality compositions of demand and output give rise to different technical progress in terms of intensity and quality bias. Moreover, the *value* of quality-biased technical progress changes along different stages of development: as individuals become richer and consume higher quality, quality-biased technical progress becomes more valuable for GDP growth. In this context, distortionary consumption taxes (or subsidies) can be used to influence the quantity/quality composition of demand in order to enhance growth by appropriately directing technical progress.

**Influencing Demand Composition** At any point in time, the government can influence the instantaneous equilibrium pair  $(x_t, z_t)$  by using a non-linear tax/subsidy scheme. Consider the following tax/subsidy scheme parameterized by  $\tau > 0$ :  $p_{\tau,t}(q) = [p_t(q)]^\tau$ ; where  $p_{\tau,t}(q)$  is the after-tax/subsidy price of quality  $q$  at time  $t$ . Clearly,  $\tau > 1$  (respectively,  $\tau < 1$ ) involves a tax/subsidy scheme relatively unfavorable (respectively, favorable) to higher-quality goods. Consumers are assumed to finance these subsidies or receive the yields of this tax scheme in a lump sum. The production function in (19) together with perfect competition yields  $p_{\tau,t}(q) = \eta [w_t e^{q\alpha/\gamma_t} / A_t]^\tau$ . Plugging in after-tax prices into the income constraint and maximizing utility yields a new version of expression (20):

$$z_t^\tau = \frac{x_t}{\sigma - x_t}. \quad (24)$$

This expression together with (19) determines the new instantaneous equilibrium. Now, taking derivatives in (24) yields:

$$\frac{dz_t}{d\tau} = \frac{x_t [\sigma - x_t]}{\sigma [\partial x_t / \partial z_t] - \tau [\sigma - x_t]^2} < 0; \quad (25)$$

where from (19) we have  $\partial x_t / \partial z_t = -x_t$ . Therefore, as expected, tax/subsidy schemes that tax higher-quality goods at higher rates (or subsidize them at lower rates) reduce the quality of consumption and increase quantity. This can be used to affect technical progress and growth.

**Enhancing Growth** At early development stages, the small scale of output may be the most important impediment for faster technical progress. Therefore, reducing the demand

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<sup>21</sup>See for example Frank (2005) and references therein.

for quality and raising the scale of output may enhance GDP growth. However, promoting quality-biased technical progress may be optimal at later stages of development when consumers are richer and efficiency in producing higher quality becomes more valuable. This is in fact the case in this model. To see this, consider the impact of a tax/subsidy scheme  $\tau$  on GDP growth. From (21), we have:

$$\frac{dg(y_t)}{d\tau} = \left[ \frac{dg(A_t)}{dz_t} + z_t \frac{dg(\gamma_t)}{dz_t} + g(\gamma_t) \right] \frac{dz_t}{d\tau}. \quad (26)$$

Changing  $\tau$  affects GDP growth through three components: the growth of  $A_t$ , the growth of  $\gamma_t$ , and the GDP-growth impact of  $\gamma_t$  growth which is given by  $z_t$ . As shown in (25), higher  $\tau$  reduces the quality of output and increases its scale. This is always positive for  $g(A_t)$  but may have an uncertain effect on  $g(\gamma_t)$ , and has a negative effect on  $z_t$ . The relative importance of these three effects changes over time. Early stages of development are characterized by a low general efficiency  $A_t$ , resulting in an output consumption  $x_t$  close to zero and minimal quality. Increasing the scale of output at these stages brings about a sizable increase in general efficiency  $A$  growth, whereas the alternative of increasing quality would not bring about much quality-biased technical progress because this also requires a large scale of output, which is hurt by increasing quality (see (22) and (23)). Moreover, quality-biased technical progress is of little value because individuals are consuming very low quality at these stages (hence  $z_t$  is low in (21)). Hence, a tax/subsidy scheme that shifts GDP towards larger output has a positive net effect on GDP growth at early development stages. However, the positive effect of a progressive tax scheme on  $g(A_t)$  becomes weaker at later stages, as  $A_t$  increases. Meanwhile, the negative effect on  $g(\gamma_t)$  becomes larger, while the growth value of being more efficient in producing higher quality (which is given by  $z_t$ ) also increases. Therefore, for some parameter values, a tax/subsidy scheme that shifts consumption towards higher quality and lower quantity may have a positive effect on GDP growth at more advanced stages of development.

**Proposition 6** *At early stages of development, progressive consumption taxes enhance GDP growth. This may not be the case at later stages.*

**Proof.** Consider the initial stages of development, which are characterized by  $A_t$  being close to 0. As already noted, from (19)-(20) we have  $\lim_{A_t \rightarrow 0} x_t = \lim_{A_t \rightarrow 0} z_t = 0$ . Now consider the three components in (26). From (22) we have  $\lim_{A_t \rightarrow 0} dg(A_t)/dz_t = -\theta_1/e^{z_t} < 0$ ; whereas

from (23) we have  $\lim_{A_t \rightarrow 0} dg(\gamma_t)/dz_t = 0$  and  $\lim_{A_t \rightarrow 0} g(\gamma_t) \leq 0$ . Since  $dz_t/d\tau < 0$  (see (25)), this yields  $\lim_{A_t \rightarrow 0} dg(y_t)/d\tau > 0$ . Now, continuity of these derivatives implies that at least for an initial interval of time such that  $A_t$  is still sufficiently low, GDP growth is increasing in the tax scheme  $\tau$ . ■

Note that since the arguments in the proof do not use the assumptions on preferences, this proposition holds for any utility function as long as  $dz_t/d\tau < 0$ . As already noted, once output has reached a sufficiently large scale, growth could be more effectively enhanced by favoring the quality dimension of consumption. That is, for some parameters of the model, a subsidy to the production of higher quality goods,  $\tau < 1$ , may enhance growth. This would be the case, for example, for  $\delta_1$  and  $\delta_2$  close to zero and  $\ln(\theta_1/\delta_1) < 2$ .<sup>22</sup> This, of course, requires  $\theta_2$  being strictly positive. In the case  $\theta_2 = 0$ , which corresponds to a standard learning-by-doing model where technical progress is unbiased, progressive taxation always enhances growth. The obvious reason is that, in this case, only the quantity of output has a positive effect on technical progress.

## 7 Inequality and Growth: The Quality Channel

This section analyzes the impact of income inequality on growth channeled through the quantity/quality composition of output. In this model, the distribution of income affects the quantity/quality output composition, which in turn influences the intensity and quality bias of technical progress. This in turn affects growth. Given the technology at a point in time, higher inequality involves higher average quality of output but smaller scale. Since at early stages of development a larger scale tends to be more beneficial to growth than higher average quality, higher inequality hinders growth during these stages. Nonetheless, the sign of this effect can change at later stages: when the scale of production is already large and when consumers prefer upgrading the quality of consumption rather than augmenting its quantity, increasing the quality bias of technical progress may help GDP growth. As a result, higher inequality, which raises the quality bias of technical progress, may have a positive impact on growth.

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<sup>22</sup>In such a case, we have  $z^* < 2$ . The argument is completed using (26) to get  $dg(y_t)/d\tau = \left[ -\frac{\theta_1}{e^{z_t}} + \theta_2 \sigma \frac{z_t^2}{1+z_t} (2 - z_t) - \delta_2 \right] \frac{dz_t}{d\tau}$ , substituting with  $\theta_1/e^{z^*} = \delta_1$ , and applying continuity around the steady state.



## 7.1 The Model with Income-Heterogeneous Agents

Income inequality can be introduced in a simple way by assuming that different individuals supply different *efficiency units of labor*. Let  $\ell_i$  be individual  $i$ 's inelastic supply of efficiency units of labor;  $i = 1, \dots, I$ . Efficiency units of labor are normalized so that total supply is equal to one:  $\sum_i \ell_i \equiv L = 1$ . All individuals have the same utility function (17) assumed in the previous two sections. I adopt the strong simplification that every period each individual's consumption is equal to her income. Technology is also the same as in previous sections, except that now labor in the production function is measured in efficiency units.

Each efficiency unit of labor earns wage  $w_t$  at time  $t$ . GDP is denoted by  $Y_t$ . Since  $L = 1$ , we have  $w_t = Y_t$ . Different individuals may now consume different qualities since their incomes may be different. Denoting by  $x_{qt}$  the production of quality  $q$  at time  $t$ , GDP is now given by:

$$Y_t \equiv \sum_q x_{qt} p_t(q). \quad (27)$$

Denote individual  $i$ 's income by  $y_{it}$  and her share in aggregate income by  $\beta_i$ :  $y_{it} \equiv \ell_i w_t = \beta_i Y_t$ . Individual  $i$ 's quantity of consumption at time  $t$  is  $x_{it}$ ,  $q_{it}$  is its quality, and  $z_{it} \equiv q_{it}/\gamma_t$ . Given  $A_t$ , individual  $i$ 's budget constraint at time  $t$  is  $\beta_i Y_t = x_{it} p_t(q_{it}) = x_{it} \frac{e^{z_{it}}}{A_t} w_t$ . Thus, since  $w_t = Y_t$ , individual  $i$ 's attainable pairs satisfy:

$$x_{it} = \frac{A_t}{e^{z_{it}}} \beta_i. \quad (28)$$

Intersection of (28) with expression (20) from utility maximization determines individual  $i$ 's equilibrium at time  $t$  (which, in turn, given  $\gamma_t$ , determines her quality choice  $q_{it}$ ). See Figure 5 where (28) is drawn for two individuals  $m$  and  $n$  such that  $\beta_m > \beta_n$ . Clearly,  $\beta_m > \beta_n$  implies  $(x_{mt}, z_{mt}) > (x_{nt}, z_{nt})$ .

Using (28) to substitute in (27) yields  $Y_t = \sum_i \frac{A_t}{e^{q_{it}/\gamma_t}} \beta_i p_t(q_i)$ . Hence, GDP growth at constant prices is given by:

$$g(Y_t) = g(A_t) + g(\gamma_t) \sum_i \beta_i z_{it}. \quad (29)$$

Finally, expressions (22) and (23) become:

$$g_{A_t} = \frac{\theta_1}{A_t} \sum_i x_{it} - \delta_1; \quad (30)$$

$$g_{\gamma_t} = \frac{\theta_2}{\gamma_t} \sum_i x_{it} q_{it} - \delta_2. \quad (31)$$

## 7.2 Inequality and Growth

The growth impact of inequality is analyzed by considering the instantaneous growth effect of a change in the distribution of the  $\beta_i$ s (with  $\sum_i \beta_i = 1$ ), with given technology and aggregate labor supply.<sup>23</sup> Specifically, I consider the impact of higher inequality defined as a mean-preserving spread on the distribution of the  $\beta_i$ s; i.e., a redistribution from individuals with lower  $\beta_i$  to individuals with higher  $\beta_i$ . This involves analyzing the sign of  $\frac{dg(Y_t)}{d\beta_m} - \frac{dg(Y_t)}{d\beta_n}$ , for  $\beta_m > \beta_n$ . In turn, this accounts to analyzing the sign of  $\frac{d^2g(Y_t)}{d\beta_i^2}$ , where a negative value would imply a negative impact of inequality on growth.

From (29) the derivative of GDP growth with respect to  $\beta_i$  is given by

$$\frac{dg(Y_t)}{d\beta_i} = \frac{dg(A_t)}{d\beta_i} + \frac{dg(\gamma_t)}{d\beta_i} \sum_i \beta_i z_{it} + g(\gamma_t) \frac{d \sum_i \beta_i z_{it}}{d\beta_i}. \quad (32)$$

In this expression,  $g(A_t)$  is increasing and concave in  $\beta_i$ . The reason is that  $g(A_t)$  only depends on the sum of quantities being produced  $\sum_i x_{it}$  and consumption quantities are concave in income. Thus, increasing inequality is negative for  $g(A_t)$ . In turn,  $g(\gamma_t)$  (which depends on  $\sum_i q_{it} x_{it}$ ) is increasing in  $\beta_i$  and initially convex, but can change to concave before the steady state is reached. Similarly,  $\sum_i \beta_i z_{it}$  is also convex in  $\beta_i$  for low  $z_{it}$ . Hence the sign of the relationship between inequality and growth through the quantity/quality mechanism may change over time as a country develops.

However, inequality has a definite negative effect at early stages of development. The reason is that, at these stages, the *growth value* of  $g(\gamma_t)$  (which is given by  $\sum_i \beta_i z_{it}$ ; see the second right-hand term of expression (32)) tends to be null because the preference for quality at low income levels is very low (note that both  $x_{it}$  and  $z_{it}$  are close to zero when  $A_t$  is close to zero, even for the richer individuals). In other words, being able to produce higher quality at low cost has little value at early stages of development. Moreover,  $g(\gamma_t)$  in the last term of (32) is also close to zero since the  $x_{it} z_{it}$  terms are close to zero. Therefore, only the negative effect of inequality on  $g(A_t)$  is quantitatively relevant at early stages. Conversely, at more advanced stages of development, the effect on  $g(A_t)$  becomes small whereas the

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<sup>23</sup>Note that  $\beta_i$  may be seen as the share of the aggregate labor supply (in efficiency units) that individual  $i$  is able to buy in order to produce her consumption bundle. The analysis may be seen as comparing growth for two different economies with the same technology and aggregate labor endowments but different distribution of efficiency units of labor, or as considering the growth effect of an income redistribution through a government lump-sum tax/transfer scheme.

value of raising quality efficiency  $\gamma_t$  increases. Therefore, the sign of the inequality/growth relationship may change to positive depending on the specific value of the parameters.<sup>24</sup>

**Proposition 7** *At early stages of development, inequality has a negative impact on growth through the quantity/quality mechanism. The sign of this impact may change at later stages.*

**Proof.** See Appendix. ■

Another intuitive way to explain this proposition is as follows. More unequal poor economies spend a larger share of GDP in the production of luxury goods consumed by the elite. This limits the possibilities of generating aggregate productivity gains for two reasons. First, achieving significant technical progress through learning-by-doing requires a large scale of production. However, producing for the elite in a poor country implies that the scale of production is small. And second, technical progress will tend to be strongly quality biased. However, technical progress in producing higher-quality goods may be of little value for producing the varieties consumed by the majority of the population (who, even in poor unequal societies, absorb a large share of GDP).

As pointed out in the Introduction, the prediction in this proposition is consistent with most empirical evidence on the relationship between inequality and growth. Work on the relationship between inequality and growth tends to be impaired by reverse causality problems and the lack of reliable data. To reduce these problems, Easterly (2007) instruments for inequality using some physical characteristics of a country's land. He finds an overall negative impact on growth. Still, there is evidence suggesting that this impact may be non-linear and may even turn positive for the group of richer countries. In fact, Deininger and Squire (1998) found that inequality reduces income growth in poor countries but not in rich ones. In turn, Forbes (2000) found a positive relationship between inequality and growth using a sample that included mostly developed countries. This non-linear effect between inequality and growth may be explained by other mechanisms different from the quality/quantity mechanism explored here. For example, it may be explained by the negative consequences of credit constraints on human capital (Galor and Zeira 1993), as long as human capital is the relatively more scarce production factor in poor countries and these constraints are loosened

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<sup>24</sup>Clearly, if  $\theta_2 = \delta_2 = 0$ , the quality being produced has no impact on technical progress. Hence, growth is always maximized by maximizing quantity at every point in time, which in turn implies that inequality is always negative for growth.

as countries become richer. However, Barro (2000) and (2008) still finds a non-linear effect of inequality on growth after controlling for schooling as well as other possible mechanisms suggested by the literature (such as the investment ratio and rule of law). Specifically, he finds a negative impact of inequality on growth among the group of less developed countries, which vanishes or even turns into a positive impact among the group of richer countries, as predicted by the model in the paper.

## 8 Concluding Comments

Recent empirical work in several areas such as real GDP growth measurement and international trade has uncovered the increasing importance of the quality dimension of output. This paper explores the consumer time-constraint foundations of the demand for quality and some of its macroeconomic implications. The starting point is that consumption requires time and that higher-quality goods provide higher utility per unit of time though at a higher monetary cost. It is then shown that additions to income are increasingly spent upgrading the quality of consumption. The paper provides a very tractable model where macroeconomic issues related to the quality dimension of goods can be analyzed. Four topics receive a first-bite examination: the share of quality growth as a component of GDP growth, the quality bias of technical progress, the role of progressive consumption taxes as a growth policy, and the impact of inequality on growth through the quantity/quality mechanism. In spite of its simple structure, the growth model in this paper can deliver reasonable quantitative paths for the secular growth of GDP in advanced economies. Notwithstanding, the calibration put forward seems to largely overestimate the quality component of GDP growth. Building and calibrating models that can match the quantity and quality components of long run GDP growth –as more thorough estimates of these components become available– and that can be linked to recent patterns of international trade and technical progress seems an interesting area for future research.

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## 9 Appendix: Proofs of Propositions

### 9.1 Proof of Proposition 5

First consider the instantaneous equilibrium at initial stages of development (i.e., for  $A(t)$  close to zero). Note that  $\lim_{A_t \rightarrow 0} g(A_t) = \theta_1 - \delta_1 > 0$ . In turn,  $\lim_{A_t \rightarrow 0} z_t = 0$  and  $\lim_{A_t \rightarrow 0} x_t = 0$ ; and therefore,  $\lim_{A_t \rightarrow 0} g(\gamma_t) = \lim_{A_t \rightarrow 0} \theta_2 z_t x_t - \delta_2 \leq 0$ . Also note that growth rates are continuous in time. Hence, for  $A(t)$  sufficiently close to zero we have  $g(A_t) > g(\gamma_t)$ . Therefore, at early stages of development, quality-neutral technical progress  $g_A$  is the main source of growth.

Now consider later stages of development. Recall that  $z_t$  increases monotonically to its steady state. Hence expression (22) implies that  $g(A_t)$  decreases monotonically from  $g_A(0) > 0$  to  $g_A^* = 0$ . On the other hand, since  $z_t$  and  $x_t$  increase monotonically towards their steady state values, (23) implies that  $z_t g(\gamma_t)$  increases monotonically towards  $z_t^* g_\gamma^*$  which is strictly positive.

### 9.2 Proof of Proposition 7

Early stages of development are characterized by low levels of general efficiency  $A$ . Furthermore, increases in inequality are defined as income redistributions from individuals with



lower  $\beta_i$  to individuals with higher  $\beta_i$ . Thus, to prove Proposition 7 I will show that for  $A$  sufficiently small,  $d^2g(Y_t)/d\beta_i^2$  in expression (29) is negative.

Before turning to that derivative, first consider the derivatives of  $x_{it}$ ,  $z_{it}$ , and  $x_{it}z_{it}$  with respect to  $\beta_i$ . The time subscript does not play any role in the following and therefore it is suppressed. Given  $A$ ,  $x_i$  and  $z_i$  can be solved from the system (20)-(28). Using this system (and taking into account that in equilibrium  $x_i = \sigma \frac{z_i}{1+z_i}$ ), the derivatives of  $x_i$  and  $z_i$  with respect to  $\beta_i$  yield

$$\begin{aligned} \frac{dx_i}{d\beta_i} &= \frac{A}{e^{z_i}} \frac{(\sigma - x_i)^2}{(\sigma - x_i)^2 + \sigma x_i} > 0, & \frac{d^2x_i}{d\beta_i^2} &= -\frac{A}{e^{z_i}} \frac{(z_i + z_i^2 + 1)(z_i + 2)(z_i + 1)\sigma^3}{(\sigma + z_i\sigma + z_i^2)^3} \frac{dz_i}{d\beta_i} < 0; \\ \frac{dz_i}{d\beta_i} &= \frac{A}{e^{z_i}} \frac{(z_i + 1)^2}{(z_i + z_i^2 + 1)\sigma} > 0, & \frac{d^2z_i}{d\beta_i^2} &= -\frac{A}{e^{z_i}} \frac{(2z_i + z_i^2 + 3)(z_i + 1)z_i}{(z_i + z_i^2 + 1)^2\sigma} \frac{dz_i}{d\beta_i} > 0. \\ \frac{d(x_i z_i)}{d\beta_i} &= \sigma \frac{(z_i + 2)z_i}{(z_i + 1)^2} \frac{dz_i}{d\beta_i} > 0, & \frac{d^2(x_i z_i)}{d\beta_i^2} &= -\frac{A}{e^{z_i}} \frac{(z_i^3 + 2(z_i^2 + z_i - 1))(z_i + 1)}{(z_i + z_i^2 + 1)^2} \frac{dz_i}{d\beta_i}. \end{aligned}$$

The sign of the last expression depends on the size of  $z_i$ . Now, consider the second derivative of expression (29) with respect to  $\beta_i$ :

$$\frac{d^2g(Y_t)}{d\beta_i^2} = \frac{d^2g(A)}{d\beta_i^2} + \frac{d^2g(\gamma)}{d\beta_i^2} \sum_i \beta_i z_i + 2 \frac{dg(\gamma)}{d\beta_i} \left[ \beta_i \frac{dz_i}{d\beta_i} + z_i \right] + g(\gamma) \left[ \beta_i \frac{d^2z_i}{d\beta_i^2} + 2 \frac{dz_i}{d\beta_i} \right] = H \frac{dz_i}{d\beta_i};$$

where

$$\begin{aligned} H \equiv & -\theta_1 \frac{1}{e^{z_i}} \frac{(z_i + z_i^2 + 1)(z_i + 2)(z_i + 1)\sigma^3}{(\sigma + z_i\sigma + z_i^2)^3} + \theta_2 \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i \\ & + 2\theta_2 \sigma z_i \frac{2 + z_i}{(1 + z_i)^2} \left[ \beta_i \frac{dz_i}{d\beta_i} + z_i \right] + g(\gamma) \left[ \beta_i \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) + 2 \right] \end{aligned}$$

Taking into account that  $\lim_{A \rightarrow 0} x_i = \lim_{A \rightarrow 0} z_i = 0$ , we have

$$\lim_{A \rightarrow 0} H = -2(\theta_1 + \delta_2) < 0.$$

Note that  $x_i$ ,  $z_i$  and all the derivatives above are continuous in  $A$ . Therefore, for  $A$  sufficiently close to 0 we have  $d^2g(Y_t)/d\beta_i^2 < 0$ .

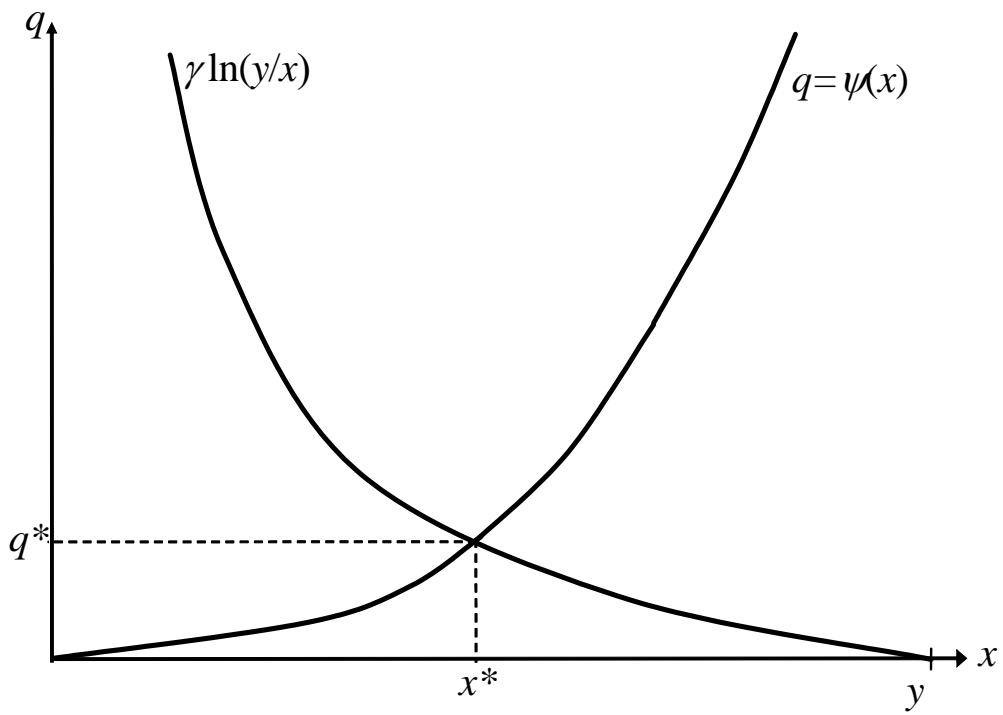


Figure 1

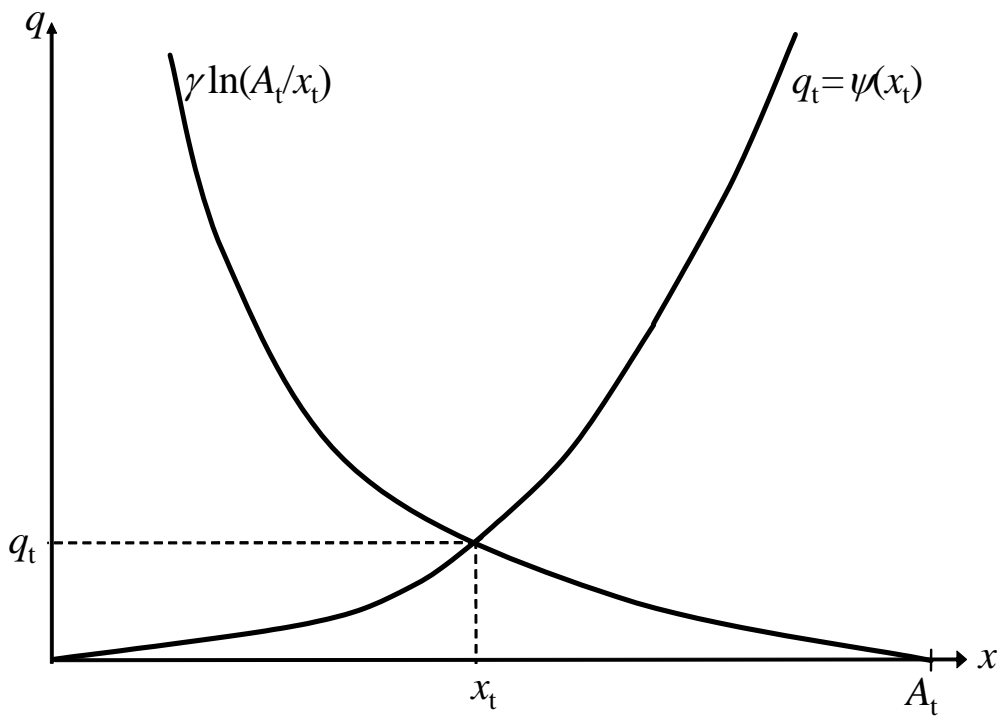


Figure 2

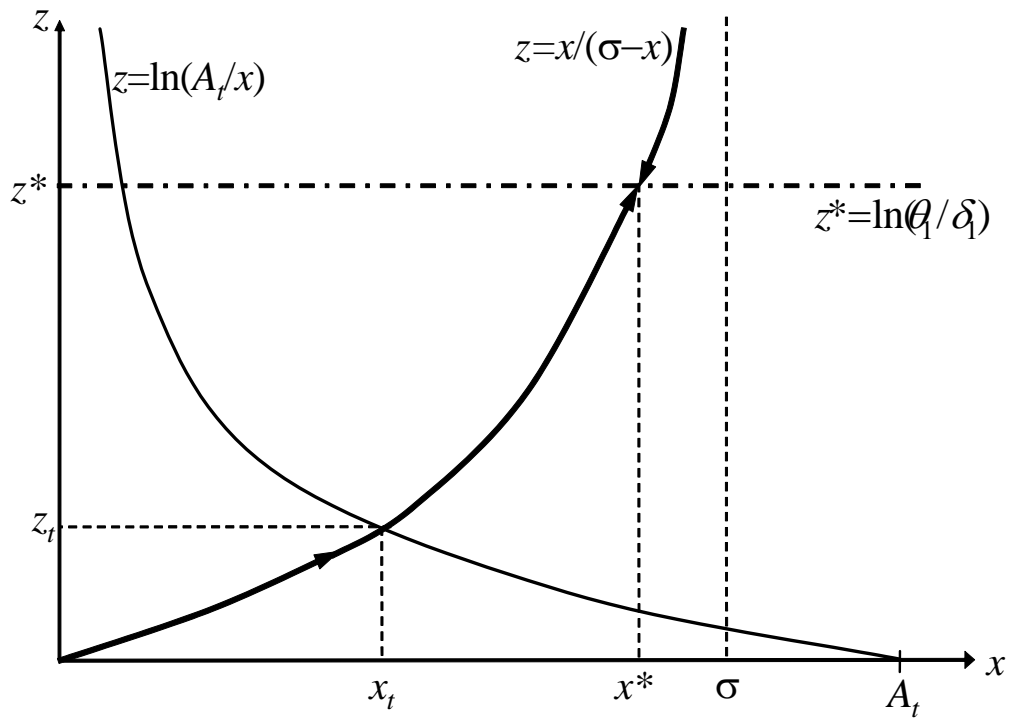


Figure 3

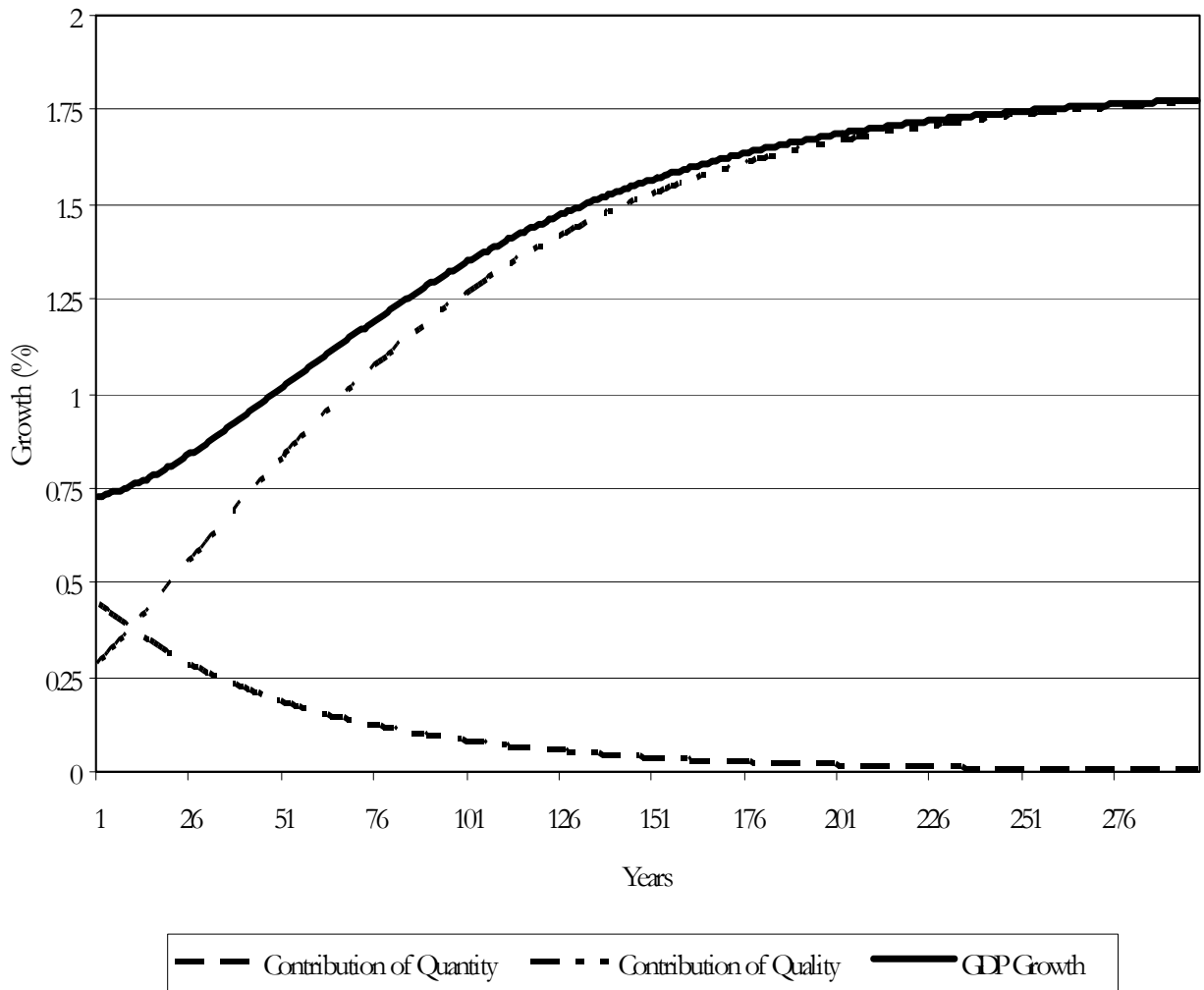


Figure 4

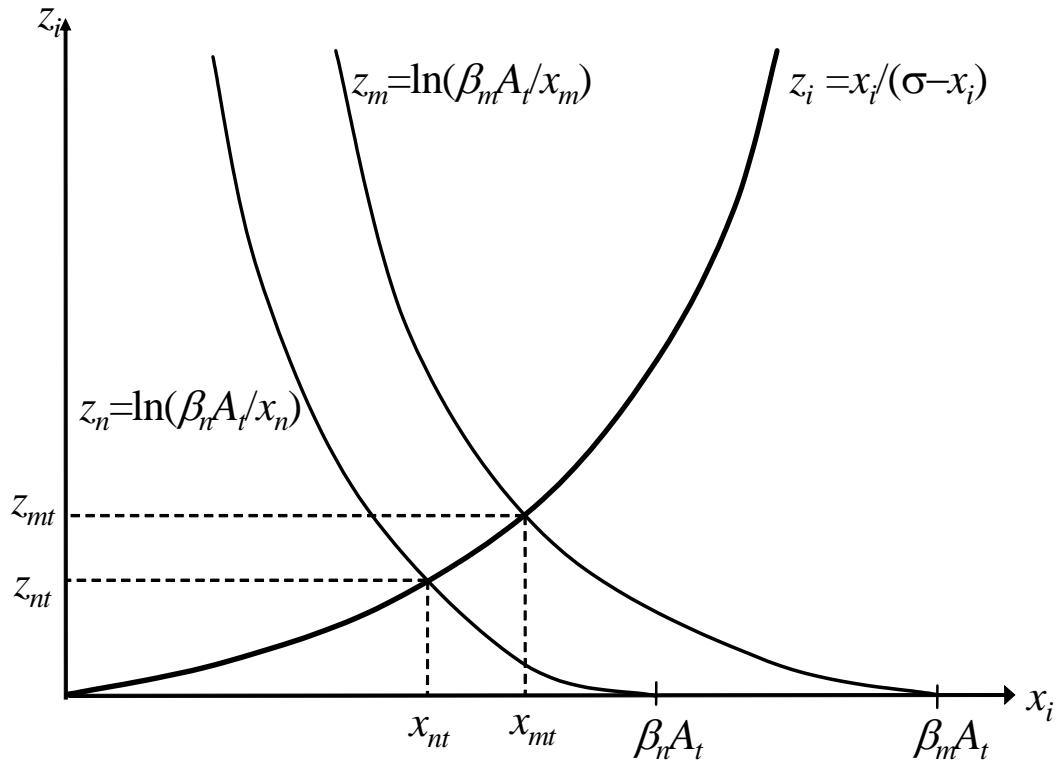


Figure 5

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**Departamento de Fundamentos del Análisis Económico**

Facultad de Economía y Empresa

E-30100 Campus de Espinardo

Murcia (SPAIN)

Tel.: (34) 868 883 784

E-mail: [nsb@um.es](mailto:nsb@um.es)

Website: [www.um.es/analisiseco](http://www.um.es/analisiseco)

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