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Abstract

In this paper we model an overlapping generation economy affected by an unexpected immigration shock and figure out how households would insure against "immigration risks" efficiently. We use the model to study the impact of immigrations on *(i)* the welfare of different generations, *(ii)* the distributions of income among factors of production, and *(iii)* the optimal design of the intergenerational welfare state. In particular, we construct a system of public education and public pensions that mimics the efficient complete market allocation. We also show the impact of immigration shocks in a small open economy. In this case the external capital flows can act as substitutes for the missing private insurance markets. Our analysis delivers a set of predictions that we find useful to understand some aspects of the Spanish experience during 1996 and 2007.

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1. Introduction

The question we are interested in is the following: what are the intergenerational economic effects of a large immigration flow? How households would insurance against an immigration shock? How does it affect the welfare of the different generations in the receiving country, both those alive and those not yet born? In particular, how does immigration impact on intergenerational arrangements such as public education and pensions, which make up the core of the contemporary welfare state? To begin answering these questions we develop a simple theoretical framework with overlapping generations that live for three periods, accumulate human capital in the first, work in the second and retire in the third living off the return from their investments. The latter include both physical capital and the resources they lent the young people to invest in human capital as we allow for this kind of lending-borrowing relations to be established, through financial markets, in our baseline model.

We take the immigration shock to be an increase in the size of the middle-age generation engendering, among other things, a reduction in the average human capital of the labor force. The immigrants are, in other words, new middle age workers somewhat less skilled than the average native one. The shock lasts one period, after which the economy moves along its new growth path with a larger number of middle age workers. We assume that the children of the immigrants perfectly integrate, hence after one period they accumulate as much per-capita human capital as the offsprings of the native workers with the same level of skill. One should note that, in the context of our model, one period lasts about 25-30 years.

Because we are interested in figuring out how, if they could, households would insure against the "immigration risk" we assume financial markets are sequentially complete in the baseline model. Because there are always two possible states of the world next period - one with and one without immigration - there are two financial assets agents buy from, and sell to, each other in every period. One asset pays one unit of consumption only when there is an immigration shock while the other pays a unit of consumption only when there is no immigration shock. Through these two assets - accessible to all individuals living in the country - young and middle age people insure themselves from the impact of an immigration shock. In particular: young people, who will be middle age and working next period, would like to insure against the negative impact that the arrival of immigrants may have on their net wages; they do so by purchasing insurance from the currently middle age people. The latter - who are saving for retirement - can use the extra payoff they would receive from their capital investment if, next period, the immigration shock were realized, to provide such insurance. Old people do not accumulate further assets, as we assume that they must die without debt and the bequest motive is absent.

The buying and selling of insurance takes place at the same time and through the same instruments that middle age and young use to lend/borrow to/from each other. More precisely: middle age individuals invest in physical capital (by purchasing assets issued by the competitive firms carrying out production next period) and in human capital (by purchasing assets issued by the young agents to finance their own education). Because, when there is immigration, the capital invested in the firms pays off more, it compensates for the lower payoff from the human capital investment accruing to the middle age. This assures that, in the benchmark complete markets economy, both young and middle age people implement as much consumption smoothing as it is feasible - consumption taking place when, respectively, they are middle age and old.

This does not imply perfect consumption smoothing, nor that some ex-ante notion of efficiency is satisfied at the equilibrium of our benchmark model. This is because agents cannot insure beforehand against the risk of being born in a period of high immigration. This is a feature of the world that is well captured by OLG models. Young agents with low skill level born in a period with a positive immigration shock are worse off than they would be otherwise, as they must compete with the offsprings of the immigrants both to borrow funds for investing in human capital this period, and in supplying labor to the market next period. This type of risk cannot be insured away either, we assume. It would be insurable if parents were altruistic and internalized, via bequests, the future welfare of their children. We assume, instead, that parents are selfish and do not leave anything to their children, hence the latter must bear the cost of being born in the "wrong" period. The extension to the case in which a bequest motive leads parents to purchase insurance for the future generations is an interesting venue for future research.

The key channels through which immigration affects welfare in this economy is that it increases labor supply of unskilled workers in the face of a predetermined stock of physical capital and skilled workers. This lowers wages of unskilled workers and increases both the return on physical capital and wages of skilled workers, shifting income from one part of the population to another. In this sense, factor prices move around because, in the benchmark model, we have assumed there is zero mobility of both physical capital and skilled labor. If there were perfect mobility of capital and skilled labor, both factors of production would flow into the country from outside on the footsteps of unskilled immigrant labor, and the capital intensity ratios would remain unchanged. In this case factor prices would be unaffected by immigration, which would amount to nothing more than an increase in the size of the economy. Under constant returns to scale in production, which we assume, this does not affect the welfare of the native agents. The capital intensity ratios remain constant, so does the wage per unit of human capital, hence the salaries of the native do not change at all. This case is trivial, hence we do not consider it.

Nevertheless, if there are frictions in the international financial markets and capital adjustment is not instantaneous, i.e. it takes time for the capital stock of the country to be built up to restore the initial capital intensity ratio, then immigration causes a redistribution between generations, as outlined above. The latter observation suggests that, the larger is the trade deficit following an immigration shock, the quicker will be the adjustment toward the old capital intensity ratio, hence the smaller the redistribution away from native workers and toward native owners of capital. This is an interesting result as it suggests that trade deficit following an immigration shock and borrowing from aboard can be a substitute for the missing internal insurance markets.

We ask next whether government policies can be used to substitute for the credit and insurance markets of the baseline model when these, as it is often the case in reality, are either absent or largely incomplete. To do this we build on previous results presented in Boldrin and Montes (2005) - which answered the question in the affirmative for the case of no immigration shock - adapting their framework to the particular circumstances at hand. In the present case we show that pension payments and social security contributions must be negatively indexed to the size of the immigration flow, while educational expenditures and the issuance of public debt financing should be positively correlated. Intuitively, this is because social security contributions play the role that the repayment of debt plus interest - by the currently middle age generation to those that lend them money to invest in human capital - plays in the model with sequentially complete financial markets. The pension payments are nothing but these contributions as received by the old people: they correspond to the payoff from the securities that were traded to finance the human capital investment of the young generation in the previous period. Likewise, the educational investment (financed via the issuance of bonds) corresponds to the issuance of the same securities in this period, hence it should increase as the size of the young generation is larger than expected.

Other authors (Shiller (1999), Bohn (1998,1999)) have stressed the positive role of an unfunded social security system as an instrument to efficiently reallocate the economic impact of aggregate shocks across different generations. They argued that, if the returns to capital and wages are imperfectly correlated and driven by an aggregate shock, an unfunded social security system that endows retired households with a claim to labor income may serve as such risk sharing tool between generations. Krueger and Kubler (2005) point out that the potentially positive intergenerational risk sharing role of social security needs to be traded off against the standard crowding-out effect that unfunded social security has on private savings and thus capital formation. In a realistically calibrated economy with stochastic production, they find that the intergenerational risk sharing role of unfunded social security system is dominated in its importance by the adverse effect on physical capital accumulation arising from the introduction of such a system. Sanchez-Marcos and Sanchez (2004) confirm the findings of Krueger and Kubler (2005) for the case of demographic uncertainty.

An important difference with respect to our economy is that, in all of these papers, the authors abstract from accumulation of human capital and, therefore, from the negative effect that missing credit markets has on education. As we show in section 4 (and in more detail in Boldrin and Montes (2005)), when credit markets for education are absent, even in the presence of government financed education there is "too much" investment in physical capital respect to the complete market allocation. This is because public education allows the working generation to invest in the human capital of the future generations, but it does not allow the former investors to collect the market return from their beneficiaries. This will, generally, lead to an inefficiency: investment in physical capital is too high and there is less intergenerational consumption smoothing than under the complete market allocation. In this sense, the introduction of a PAYGO system, in which social security contributions correspond to the capitalized value of education services received, is a tool for "efficiently" crowding-out physical capital.

The rest of the paper proceeds as follows. In Section 2 we describe the benchmark model, in Section 3 we show the effects of the absence of credit and insurance markets; in Section 4 we look at the efficient welfare state, in the presence of immigration, in a close economy with incomplete markets; in Section 5 we look at an open economy with incomplete financial markets but with public education and pensions. Section 6 concludes with some practical considerations about the Spanish experience.

2. The basic model

We use an OLG model with two types of agents, living for three periods - youth, middle age and old - in each generation. Agents differ in the productive skills' level they inherit: high (H) and low (L), respectively.

There is aggregate uncertainty due to an immigration flow that may increase the size of the low skill middle age group, thereby affecting the total supply of labor, the wage rates, the return on capital, aggregate output and the size of future generations.

We use superscripts, y, m and o to denote, respectively, young, middle-age and old people and superscripts $i = H, L$ to denote high and low human capital. The population structure, in period t, is (N_t^y, N_t^m, N_t^o), with $N_t^y = N_t^{yH} + N_t^{yL}$, $N_t^m = N_t^{mH} + N_t^{mL}$ and $N_t^o = N_t^{oH} + N_t^{oL}$. Also, $N_t^{mH} = N_{t-1}^{yH}$, $N_t^{mL} = (1+z_t)N_{t-1}^{yL}$ and $N_t^{yi} = (1+n)\, N_t^{mi}$ for $i = H, L$, where $-1 < n$, while z_t is the realization of the immigration shock in period t. For simplicity, we assume that the shock z follows a two-state Markow process, with state space $Z = \{\bar{z}, 0\}$, $\bar{z} > 0$. The notation $\pi(z_{t+1}|z_t)$ denotes the probability of $z_{t+1} \in Z$, given z_t.

In each period $t = 0, 1, ...$ a new generation $N_t^y = (1+n)\, N_t^m$ is born. Each type $i = H, L$ is born with a per-capita endowment of basic knowledge, h_t^{yi}, which is an input in the production of future human capital, according to $h_{t+1}^{mi} = h(d_t^i, h_t^{yi})$. With d_t^i we denote the physical resources invested in the education of a young individual of type i in period t; we assume $h_t^{yH} > h_t^{yL}$. The function $h(d, h^y)$ is a constant returns to scale neoclassical production function. During the second period of life, individuals work and decide how much of their income to consume, how much to save, and how to allocate the latter among different financial instruments. When old, they have no decisions to make: they consume all their income, and then die. We assume agents draw utility from consumption when middle age and old. We also assume immigrants enter the country with the same human capital as the low skill middle age natives and with zero capital or financial assets. Neither consumption when young, nor leisure, nor the welfare of descendants affect lifetime utility.

Initial conditions are: K_0, for the capital stock, $(N_0^{yi}, N_0^{mi}, N_0^{oi})$ for the population, h_0^{mi} for the human capital of the middle age individuals, $A_{-1}^{yi}(0)$, $A_{-1}^{mi}(0)$ for, respectively, the portfolios of middle age and old people, and $A_{-1}^f(0)$ for that of the representative firm, which owns K_0. Finally, we assume there are no immigrants in the first period.

The preferences of an individual of type i, born in period $t-1$, are

$$E_{t-1}\left\{u(c^{mi}(z_t))+\delta E_t\left[u(c^{oi}(z_{t+1}))\right]\right\},$$

where δ is the period discount factor and E the expectation operator. The function $u:\Re_+\longrightarrow\Re$ is assumed to be strictly increasing, strictly concave and C^2.

2.1. Market structure

Normalize to one the price of output in the initial period, in which the state is $z=0$; write $p_t(z)$ for the price of output in period t and state $z\in Z$ in all subsequent periods. We assume sequentially complete financial markets, *i.e.* that - given the current state z_t and the set Z of possible future states - for all $z\in Z$ there exists a competitive market in which contingent claims $A_t(z)$ are traded, with payoff, in units of next period consumption, $b[A_t(z), z_{t+1}]=1$ if $z_{t+1}=z$, and zero otherwise. We assume agents cannot die in debt, i.e. we impose $A_t^{mi}(z)\geq 0$ for all t and z with $i=H,L$. Let $q(z,z_t)$ be the price, in units of consumption at t, of asset $A(z)$ in period t and state z_t. Notice that here, to save notation, the symbol $A_t(z)$ indicates also the number of units of that asset traded in a given period.

2.2. Firms

There is a representative firm, which uses physical capital and the two types of human capital to produce output according to $Y_t = F(K_t,H_t,L_t)$, where $H_t=h_t^{mH}N_t^{mH}$, $L_t=h_t^{mL}N_t^{mL}$ and $F(K,H,L)$ is a constant returns to scale neoclassical production function. We assume full depreciation of capital and that high skill workers are more productive than low skill workers, everything else equal, i.e. $F_H(K,X,X)>F_L(K,X,X)$. Firms last one period and own the physical capital, which they finance by issuing state-contingent securities. More specifically, in each period t the representative firm issues securities $A_t^f(z)$ at a price $q(z,z_t)$, for $z\in\{\bar{z},0\}$, with the proceedings of which they purchase K_{t+1}, used for production next period. In period $t+1$, after the realization of the shock, the firm hires workers, carries out production, pays off wages, honors its financial liabilities and then dissolves.

Let $w^i(z_t)$ be the nominal wage, in period t and state $z_t\in Z$, for an agent of type $i=H,L$. Write $w^i(z_t)/p(z_t)=\omega^i(z_t)$ and $\varphi(z_t)=p(z_t)F_K(K_t,H_t,L(z_t))$. The problem of the firm is

$$\max_{A_t^f(z_{t+1}),H_{t+1},L_{t+1}} E_t\left\{p(z_{t+1})\left[Y(z_{t+1})-\omega^H(z_{t+1})H_{t+1}-\omega^L(z_{t+1})L(z_{t+1})-A_t^f(z_{t+1})\right]\right\}$$

subject to,

$$\begin{aligned} Y(z_{t+1}) &= F\left(K_{t+1},H_{t+1},L(z_{t+1})\right) \\ K_{t+1} &= \sum_{z\in Z} q(z,z_t)A_t^f(z). \end{aligned}$$

The first order conditions for H, L and for $A^f(z)$ are

$$\omega^H(z_{t+1}) = F_H\left(K_{t+1},H_{t+1},L(z_{t+1})\right) \quad \forall\ z_{t+1}\in Z \tag{1.a}$$

$$\omega^L(z_{t+1}) = F_L\left(K_{t+1},H_{t+1},L(z_{t+1})\right) \quad \forall\ z_{t+1}\in Z \tag{1.b}$$

$$q(z,z_t) = \frac{\pi(z|z_t)p_{t+1}(z)}{\sum_{z\in Z}\pi(z|z_t)\varphi_{t+1}(z)} \quad \forall\ z\in Z. \tag{1.c}$$

2.3. Consumers

For a native agent of type $i = H, L$ born in period $t-1$ when the state is z_{t-1}, the lifetime optimization problem is

$$\max_{d^i(z_{t-1}),A_{t-1}^{yi}(z),A_t^{mi}(z)} E_{t-1}\left\{u(c^{mi}(z_t))+\delta E_t\left[u(c^{oi}(z_{t+1}))\right]\right\}$$

subject to,

$$d^i(z_{t-1})+\sum_{z\in Z} q(z,z_{t-1})A_{t-1}^{yi}(z) \le 0 \tag{2.a}$$

$$c^{mi}(z_t)+\sum_{z\in Z} q(z,z_t)A_t^{mi}(z) = \omega^i(z_t)h_t^{mi}+A_{t-1}^{yi}(z_t) \quad \forall z_t\in Z \tag{2.b}$$

$$c^{oi}(z_{t+1}) = A_t^{mi}(z_{t+1}) \qquad \forall z_{t+1}\in Z \tag{2.c}$$

$$h_t^{mi} = h\left(d^i(z_{t-1}),h_{t-1}^{yi}\right) \tag{2.d}$$

The first order conditions for the choice of $\mathcal{A}^{yi}(z_{t-1}) = \{A_{t-1}^{yi}(z)$, for all $z\in Z\}$ and $d^i(z_{t-1})$ boil down to

$$q(z,z_{t-1}) = \frac{\pi(z|z_{t-1})u'(c_t^{mi}(z))}{\sum_{z\in Z}\pi(z|z_{t-1})u'(c_t^{mi}(z))\omega_t^i(z)h_d\left(d^i(z_{t-1}),h_{t-1}^{yi}\right)} \quad \forall z\in Z \tag{3.a}$$

$$1 = \sum_{z\in Z} q(z,z_{t-1})\omega_t^i(z)h_d\left(d^i(z_{t-1}),h_{t-1}^{yi}\right). \tag{3.b}$$

For each of the $A_t^{mi}(z)$, the first order condition reads

$$q(z,z_t) = \frac{\pi(z|z_t)\delta u'(c_{t+1}^{oi}(z))}{u'(c^{mi}(z_t))} \quad \forall z \in Z. \tag{3.c}$$

For a middle age immigrant, arriving in the state of the world z_t with human capital $\bar{h}_t^m = h_t^{mL}$ and $A_{t-1}^y(z_t) = 0$, the maximization problem is:

$$\max_{\bar{A}_t^m(z)} u(\bar{c}^m(z_t)) + E_t\left[\delta u(\bar{c}^o(z_{t+1}))\right]$$

subject to,

$$\begin{aligned} \bar{c}^m(z_t) + \sum_{z\in Z} q(z,z_t)\bar{A}_t^m(z) &= \omega^L(z_t)\bar{h}_t^m && (4.\text{a}) \\ \bar{c}^o(z_{t+1}) &= \bar{A}_t^m(z_{t+1}) \quad \forall z_{t+1} \in Z. && (4.\text{b}) \end{aligned}$$

The first order conditions determining $\bar{A}^m(z_t)$ are analogous to those in (3.c):

$$q(z,z_t) = \frac{\pi(z|z_t)\delta u'(\bar{c}_{t+1}^o(z))}{u'(\bar{c}^m(z_t))} \quad \forall z \in Z. \tag{4.c}$$

2.4. Financial markets

It should be clear from the budget constraint that the net financial position of the young is non-positive (i.e. $\sum_{z\in Z} q(z,z_{t-1})A_{t-1}^{yi}(z) \leq 0$ for $i = H, L$) and that of middle age non-negative (i.e. $\sum_{z\in Z} q(z,z_t)A_t^{mi}(z) \geq 0$ for $i = H, L$ and $\sum_{z\in Z} q(z,z_t)\bar{A}_t^m(z) \geq 0$). When the latter is positive it corresponds to aggregate national saving, which is invested in the physical capital of firms and in the education of the young agents. The first order conditions for profit maximization of the representative firm imply

$$q(z,z_t) = \frac{\pi(z|z_t)p_{t+1}(z)}{\sum_{z\in Z}\pi(z|z_t)\varphi_{t+1}(z)} \text{ for each } z \in Z. \tag{1.c}$$

Multiplying (1.c) by $F_K(K_{t+1}, H_{t+1}, L_{t+1}(z))$ and aggregating over $z \in Z$ we get

$$\sum_{z\in Z} q\left(z,z_t\right) F_K(K_{t+1}, H_{t+1}, L_{t+1}(z)) = 1. \tag{5}$$

2.5. Competitive equilibrium

A *competitive equilibrium* is a mapping from the current state of the world into a distribution of quantities and prices at all t. Given an initial condition (K_0, H_0, L_0), z_0, $\left(N_0^{oi}, N_0^{mi}, N_0^{yi}\right)$, $\left(A_{-1}^{yi}(z_0), A_{-1}^{mi}(z_0), A_{-1}^{f}(z_0)\right)$, with $i = H, L$, and a sequence of exogenous basic knowledges $\left\{h_t^{yH}(z), h_t^{yL}(z)\right\}_{t=0}^{\infty}$, a competitive equilibrium is a collection of:

1. choices of the native, $\left\{d_t^i(z), c_t^{mi}(z), c_t^{oi}(z), A_t^{yi}(z), A_t^{mi}(z)\right\}_{t=0}^{\infty}$, $i = H, L$, and immigrant, $\left\{\bar{c}_t^m(z), \bar{c}_t^o(z), \bar{A}_t^m(z)\right\}_{t=0}^{\infty}$, households;
2. choices of the representative firm, $\left\{K_t(z), H_t(z), L_t(z), A_t^f(z)\right\}_{t=0}^{\infty}$;
3. prices, $\{p_t(z), q(z, z_t)\}_{t=0}^{\infty}$ and $\left\{\omega_t^H(z), \omega_t^L(z), \varphi_t(z)\right\}_{t=0}^{\infty}$;

such that for all t and $z \in Z$, the consumers and the firm maximize their payoffs and the markets clear.

In each period t and state z there are three sets of markets to clear:
i) Output market:

$$C_t^m(z) + C_t^o(z) + D_t(z) + K_{t+1}(z) = F\left(K_t, H_t, L_t(z)\right). \tag{6.a}$$

where $C_t^m(z)$ and $C_t^o(z)$ are, respectively, aggregate consumption of middle age and old in period t and state z and $D_t(z)$ is aggregate physical resources invested in education in period t and state z.
ii) Labor market:

$$\begin{aligned} H_t &= h_t^{mH} N_{t-1}^{yH}, \\ L_t(z) &= h_t^{mL}(1+z) N_{t-1}^{yL}. \end{aligned} \tag{6.b}$$

iii) Capital market:

$$\begin{aligned} \sum_{z\in Z} q(z, z_t) A_t^f(z) &= K_{t+1}, \\ A_t^f(z) &= \sum_{i=H,L} A_t^{mi}(z) N_{t-1}^{yi} + \bar{A}_t^m(z) z_t N_{t-1}^{yL} + \sum_{i=H,L} A_t^{yi}(z) N_t^{yi}, \\ \sum_{i=H,L} d^i(z_t) N_t^{yi} &= -\sum_{i=H,L}\sum_{z\in Z} q(z, z_t) A_t^{yi}(z) N_t^{yi}. \end{aligned} \tag{6.c}$$

For each state $z \in Z$, the payoff from security $A_t^f(z)$ is:

$$\begin{aligned} b[A_t^f(z), z_{t+1} &= z]A_t^f(z) = F\left(K_{t+1}, H_{t+1}, L_{t+1}\left(z\right)\right) - \omega_{t+1}^H(z)H_{t+1} - \omega_{t+1}^L(z)L_{t+1}(z) \\ &= F_K(K_{t+1}, H_{t+1}, L_{t+1}(z))K_{t+1}. \end{aligned} \qquad (6.d)$$

2.6. Numerical Evaluation

In Appendix A we study an analytical illustration of our model in which the following specification of our key functions is used: Logarithmic utility and Cobb Douglas production functions: $u(c) = \log c$, $F(K,H,L) = AK^{\alpha}H^{\theta}L^{1-\alpha-\theta}$ and $h\left(d^i, h^{yi}\right) = B(d^i)^{\beta}\left(h^{yi}\right)^{1-\beta}$.

Our main purpose in this section is consider the practical implications of an immigration shock in a world with complete markets using a numerical evaluation of the economy in Appendix A. To do this we assign reasonable values to each parameters and provide a numerical computation of the impact of an immigration shock. We compare two economies: an economy with no immigration $(z_0, z_1, z_2, ...) = (0, 0, 0, ...)$ and another with only one immigration shock $(z_0, z_1, z_2, ...) = (0, \bar{z}, 0, ...)$. We normalize $N_0^m = 1$, with $N_0^{mH} = 0.6$ and assume an annual growth rate of the population equal to 0. Recall that a period in this model is about 30 years. We assume $\bar{z} = 0.4$ and $\pi(\bar{z}|z_t) = \pi(0|z_t) = 0.5$. With respect to the production technology, α is fixed to 0.3, $\theta = 0.45$ and the scale parameter A is fixed at 1. In the human capital technology we set $\beta = 0.13$, which corresponds to an elasticity of output with respect to education of 0.058 for a high human capital worker and 0.0325 for a low human capital worker. The discount factor δ is set to match an average ratio of investment over output $I/Y = 21.9\%$. This yields a value of $\delta = 0.904$, which corresponds to an annual discount factor of 0.996641. We set the scale parameter B equal to 4.35 to obtain an annual rate of aggregate output growth equal to 3% along the balance growth path. With these parameter values we have an annual interest rate of 4.1%. The fraction of total output spent to finance education (D/Y) is equal to 6.3%, which is on the low side for the US but not for most European countries, including Spain.

Assume the same initial conditions for both economies and assume they are on their balance growth path from the start. Denote with "hat symbols" $(\widehat{x})$ the variables in the economy with an immigration shock in $t = 1$. In Table 1 and Table 2 we show the change in utility and consumption, of middle age and old, caused by an immigration shock in period $t = 1$. Notice, first, that the middle age and old generations that are alive when the shock hits, consume more in this period than in the economy with no shock, because output is much higher during that period and the insurance mechanism redistributes this extra income to both middle age and old people. The generations with low human capital born in the future, nevertheless, are worse off in the economy with a shock because insurance against the immigration shock cannot be purchased before being born and they pay the price of having low human capital income competing with them.

The immigration shock affects negatively the low human capital workers of future generations - their wages are smaller along the transition to the new balanced growth path - and positively the high human capital workers of future generations - their wages are higher along the same path. Depending on the choice of parameter values, a second effect may or may not be pushing in the same direction: immediately after the shock hits, there is "too much" L and "too little" of both H and K, relative to the balanced growth ratios. As H and K accumulates toward the desired level, to the extent that this takes place at different speeds depending on parameter values, there may be "too little" H and "too much" K, for some periods during the transition. This pushes the wage of the high human capital workers further up, and the marginal productivity of capital, instead, down relative to their levels in the economy without a shock.

It is important to note that such effects depend upon the presence, or absence, of certain financial markets that different generations may use to purchase insurance. First of all: the low human capital workers alive during the transition periods are worse off because the immigration shock took place while they were not yet born and, therefore, could not insure against it. As there are "too many" of them during the transition, their lifetime income is lower than in the absence of the shock. Second, the rapid increase in the stock of K, right after the immigration shock hits, is due to the presence of the financial assets allowing the middle age workers then alive to profit from the shock. Because they share in the bounty with the owners of physical capital, their disposable income increases, allowing them to immediately invest more in K, which surges right away. Simple calculations show that, absent the financial instruments allowing middle age workers to share in it, the extra output accruing to firms in the form of an increase in the productivity of K would go only to the owners of K, that is the old retirees. They would consume it all instead of investing part in tomorrow's stock of capital, which is instead what the middle age people due with their share. This means that, in an economy without this kind of financial assets, the rate of return on capital jumps up right after the shock hits and then slowly decreases toward its long run position. In our economy, instead, the rate of return on capital jumps up at the time of the shock and down a period later (when investment surges) to converge from below to its long run position, as high human capital is accumulated much more slowly than physical capital is. Put it differently, the hypothesis of sequentially complete financial markets has testable aggregate implications.

In Table 2 we show the effect of the shock on the annual rate of return on capital and growth rates. In Figure 1 we show the effect of the shock on wages. As argued, the immigration shock has a positive effect on the investment rate in the period in which the immigrants arrive and a temporarily negative (positive) effect on low (high) human capital labor productivity. Capital productivity increases in the period in which the immigrants arrive and then decreases and stays temporarily below its value in the economy without shocks. After the adjustment is completed, the growth rate and the marginal productivities resume their original long-run levels.

Table 1: Change in life-cycle utility caused by an immigration shock in $t = 1$

t	$\hat{U}^H_{t-1} - U^H_{t-1}$	$\hat{U}^L_{t-1} - U^L_{t-1}$
0	0.0090	0.0090
1	0.0130	0.0130
2	0.2353	-0.4052
3	0.2332	-0.4074
4	0.2326	-0.4079
5	0.2325	-0.4081
6	0.2324	-0.4081
7	0.2324	-0.4081

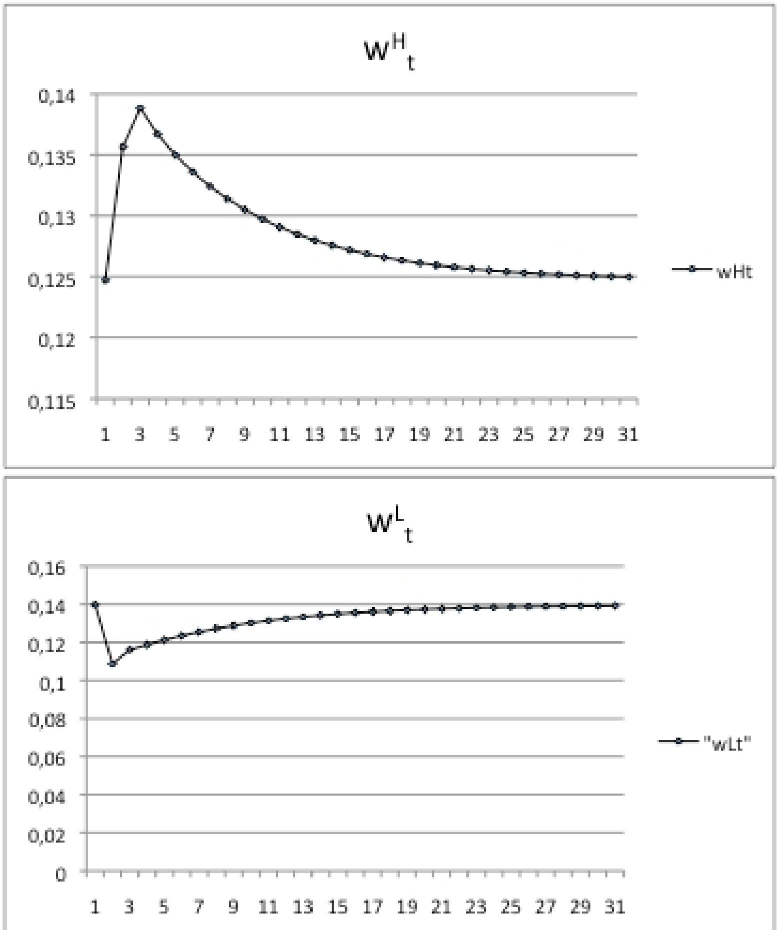
Table 2: Change in consumption caused by an immigration shock in $t = 1$.

t	$\hat{c}_t^{mH}/c_t^{mH}$	$\hat{c}_t^{mL}/c_t^{mL}$	$\hat{c}_t^{oH}/c_t^{oH}$	$\hat{c}_t^{oL}/c_t^{oL}$
0	1	1	1	1
1	1.0100	1.0100	1.0100	1.0100
2	1.1325	0.8089	1.0033	1.0033
3	1.1305	0.8075	1.1305	0.8075
4	1.1300	0.8071	1.1300	0.8071
5	1.1299	0.8070	1.1299	0.8070
6	1.1298	0.8070	1.1298	0.8070
7	1.1298	0.8070	1.1298	0.8070

Table 3 Changes in annual interest rate $\hat{r}_t$ and growth rate of GDP $\hat{g}_Y$ caused by an immigration shock in $t = 1$.

t	$\hat{r}_t$ annual	$\hat{g}_Y$ annual
0	0.04100	0.03024
1	0.04392	0.03313
2	0.04077	0.03162
3	0.04094	0.03018
4	0.04098	0.03022
5	0.04099	0.03023
6	0.04099	0.03024
7	0.04100	0.03024

Figure 1 : Changes in wages caused by an immigration shock in $t = 1$.



3. Equilibrium when credit and insurance markets are missing

The results obtained here are consistent with those we report in Boldrin and Montes (2005). Nevertheless, adding heterogeneity within each generation and uncertainty as to the size and composition of the next cohort of middle-age workers, enriches the model and makes it possible to ask a number of new, interesting question. More precisely, one would want to distinguish the study of what happens, (1) when markets to insure against unexpected immigration shocks are absent, from, (2) what happens when markets for lending/borrowing across generations and over time are not available. This leads us to consider separately the following two cases.

1. Young people cannot trade the security $A^y(z)$, still they can borrow to invest d in human capital. Because there are no state contingent assets, they must repay their debt at a fixed interest rate, no matter if next period an immigration shock is or is not realized. In other words, they can borrow but they cannot insure. In this case, even if the middle age people attempted to trade in state contingent $A^m(z)$ assets, it would not work for lack of a counterpart. The only entity they could trade those assets with is the firm, which cannot insure them against anything as it has no compensating sources of income in bad states. The income of old age people is now equal to the fixed return on d plus the random return on capital investment. Because these are linearly independent returns, and there are only two states of the world, middle age people can still use a portfolio composed of "educational bonds" and "shares of the firm" to fully insure their old age consumption (for certain parameters configuration this may require taking a negative position in one of the two assets, an impossibility in this environment). This result is special, though, as it follows from the simplifying assumption of only two immigration states, which makes spanning possible with just two assets. In either case, when insurance markets are absent the young people bear all the risk because, when middle age, they must reimburse a fixed amount $d(1+r)$, no matter what the state of the world is. This implies that, when there is immigration, low human capital middle age natives have less income to consume and save than in the complete markets case. As in the world with complete markets, their wage bill is lower but their debt payment is now higher, which leaves less for $c^{mL}(z_t) + \sum_{z \in Z} q(z, z_t) A_t^{mL}(z)$. Their lifetime utility is therefore lower. On the contrary, high human capital middle age natives have more income to consume and save than in the complete markets case (their wage bill is higher, as in the world with complete markets, but their debt payment is now lower). Also, aggregate net labor income is lower and therefore total investment decreases.

2. Young people cannot borrow at all, hence middle age people can only invest in the physical capital. Obviously this implies that there is a much lower level of human capital in the economy, and there is no growth. In this case "workers" bear all the downside risk (i.e. they either do "normal" or do "worse") whereas the owners of capital bear all the upside (i.e. they either

do "normal" or do "better").

As both these results are quite straightforward we skip the mathematical details and move on to consider if and how such inefficiencies could be alleviated with some kind of "welfare state" intervention.

4. The Welfare State of a Closed Economy

4.1. Missing credit and insurance markets

Let us begin with case 2 of the previous section, in which all credit and insurance markets other than the market for purchasing physical capital have been shut down. In this case, $F_K(K_t, H_t, L_t)$, $F_H(K_t, H_t, L_t)$ and $F_L(K_t, H_t, L_t)$ are all affected by the immigration shock and no factor owner can insure against it. We want to derive policies capable of implementing the sequentially complete market allocation (SCMA) of Section 2. They turn out not to be very different from those derived in Boldrin and Montes (2005), apart from the fact that contributions and benefits are now state contingent.

In particular, in Boldrin and Montes (2005), the young "borrow" from the middle age via the public education system and they pay back the debt, at the market interest rate, via a social security tax, the proceeding of which finance pension payments. Under uncertainty, we need to use the welfare state to also allocate risk efficiently between generations and heterogeneous agents, not just, as in the deterministic case, to allow for intergenerational trade. Think of what happens when there is an unexpected flow of immigrants ($z = \bar{z}$): the marginal productivity of low human capital labor decreases and the marginal productivity of high human capital and physical capital increase. If we simply levy a social security contribution in the amount $t_t^{pi} = d_{t-1}^{i*}(1 + r_t^*)$ where r_t is the market interest rate (starred symbols from now on refer to the SCMA quantities) and nothing else, the disposable per capita income of the low (high) human capital middle age individuals decreases (increases) compared to the SCMA. Furthermore, the amount by which the savings of this group decrease is not compensated by the increased saving of high human capital workers, implying an under-investment in physical capital compared to the SCMA.

There are, therefore, potential gains from risk sharing among high and low human capital households. We should stress here a relatively delicate point: an immigration shock causes aggregate uncertainty (it increases aggregate output) but, because it affects the three factors of production differently, part of that uncertainty is insurable. In particular, the native low human capital workers face the risk of a reduced per capita income, while the native high human capital workers and capital owners face a larger per-capita income. If there is no immigration, the opposite is true. Said it differently: as it is often the case, aggregate risk obtains from the composition of different individual risks and it has clear redistributional consequences. Some people gain and some loose even from "aggregate" shocks. Insurance, then, must work the following way: when there is immigration the old people (the owners of capital) pay something to the native middle age people, and viceversa in the other periods. Also, depending on parameter values, the native high human capital workers may or may not have to transfer something to the native low human capital workers: this will depend on how large their income gains are relative to the aggregate increase in output. To implement the SCMA the social planner, therefore, needs to emulate the way in which intergenerational insurance markets would work.

Assume a period-by-period balanced budget and introduce two tax and transfer schemes; we call the first a "pension scheme" and the second an "education scheme". Write, for each $z_t \in Z$,

$$\sum_{i=H,L} t^{pi}(z_t) N_{t-1}^{yi} + \bar{t}^{p}(z_t) z_t N_{t-1}^{yL} = \sum_{i=H,L} b^{i}(z_t) N_{t-2}^{yi} + \bar{b}(z_t) z_{t-1} N_{t-2}^{yL},$$

for the pension scheme, and

$$\sum_{i=H,L} t^{ei}(z_t) N_{t-1}^{yi} + \bar{t}^{e}(z_t) z_t N_{t-1}^{yL} = \sum_{i=H,L} e^{i}(z_t) N_{t}^{yi},$$

for the education scheme. Let us start from the last equation. Here $e^i(z_t)$ denotes the educational transfer received from each member i of the currently young generation when the aggregate shock is z_t. On the other side of the budget constraint, we find the contributions provided, respectively, by the middle age natives ($t^{ei}(z_t)$) and by the middle age immigrants ($\bar{t}^{e}(z_t)$). In the optimal policy, we treat working native differently as they receive different net income during middle age. The optimal policy also dictates treating young people differently in light of their different endowment of basic knoldwedge, without differentiating between native and immigrants.

The budget constraint for the pension scheme can be interpreted similarly, but here we need treating natives and immigrants differently on both sides. They pay different contributions ($t^{pi}(z_t)$ and $\bar{t}^p(z_t)$, respectively) and receive different benefits when retired, $b^i(z_t)$ and $\bar{b}(z_t)$. Again, this mimic what would have happened in an economy like that of section 2, when markets were dynamically complete. The important point is that, in both schemes, the contribution and benefit rates are state contingent, i.e. they change depending on the immigration flow. The latter is an aggregate variable, hence the state contingent policy does not depend on any private information but on a state variable that should, at least in principle, be observable by the policy maker.

Under these policies, the budget constraints for the representative member of the generation born in period $t-1$ become

$$
\begin{aligned}
&d^i(z_{t-1}) \leq e^i(z_{t-1}) \\
&c^{mi}(z_t) + s^i(z_t) = \omega^i(z_t) h(d^i(z_{t-1}), h_{t-1}^{yi}) - t^{ei}(z_t) - t^{pi}(z_t) \quad \forall z_t \in Z \\
&c^{oi}(z_{t+1}) = s^i(z_t) R(z_{t+1}) + b^i(z_{t+1}) \quad \forall z_{t+1} \in Z
\end{aligned}
$$

The symbol $s^i(z_t)$ is the investment in physical capital an individual of type $i = H, L$ makes in period t and state z_t, and $R(z_{t+1}) = (1 + r(z_{t+1}))$ is the return factor on saving in state z_{t+1}.

For an immigrant arriving in period t, the budget constraints read

$$
\begin{aligned}
\bar{c}^m(z_t) + \bar{s}(z_t) &= \omega^L(z_t) \bar{h}_t^m - \bar{t}^e(z_t) - \bar{t}^p(z_t), \\
\bar{c}^o(z_{t+1}) &= \bar{s}(z_t) R(z_{t+1}) + \bar{b}(z_{t+1}) \quad \forall z_{t+1} \in Z.
\end{aligned}
$$

If we set $e^i(z_{t-1}) = d^{i*}(z_{t-1})$ (starred symbols refer to the SCMA), *i.e.* we transfer educational resources to the young generation up to the point at which the expected return on education is equal to the expected return on physical capital,

$$
\sum_{z \in Z} \pi(z|z_{t-1}) p_t(z) R_t(z) = \sum_{z \in Z} \pi(z|z_{t-1}) p_t(z) \omega_t^i(z) h_d\left(d^i(z_{t-1}), h_{t-1}^{yi}\right),
$$

we reach the efficient level of human capital in period t. In Boldrin and Montes (2005) we show (in a world with no immigration shocks and with homogenous agents) that in a deterministic world this policy, together with $t^{pi}(z_t) = d_{t-1}^{i*} R^*(z_t)$ and $b^i(z_t) = t_{t-1}^{ei} R^*(z_t)$ implements the efficient CMA overall. Pension benefits received (social security contributions paid) should correspond to the capitalized value of the lifetime contributions to aggregate human capital accumulation paid (educational services received). But this policy is not enough to implement the appropriate amount of intergenerational risk sharing when random shocks affect the size of the working population. We need to add a second mechanism, allocating risk between generations.

Comparison of the last budget restrictions with the budget restrictions of the SCMA, $(2.a)-(2.d)$, shows that, if the lump-sum tax-transfer amounts satisfy

$$t^{pi}(z_t) = -A^{yi*}_{t-1}(z_t), \qquad \bar{t}^p(z_t) = 0,$$

$$\begin{aligned} b^i(z_{t+1}) &= A^{mi*}_t(z_{t+1}) - \lambda^{i*}(z_t)K^*_{t+1}R^*(z_{t+1}), \\ \bar{b}(z_{t+1}) &= \bar{A}^{m*}_t(z_{t+1}) - \bar{\lambda}^*(z_t)K^*_{t+1}R^*(z_{t+1}), \end{aligned}$$

and

$$t^{ei}(z_t) = \tilde{A}^{mi*}(z_t) - \lambda^{i*}(z_t)K^*_{t+1}, \quad \bar{t}^e(z_t) = \widetilde{\bar{A}}^{m*}(z_t) - \bar{\lambda}^*(z_t)K^*_{t+1},$$

where $\lambda^{i*}(z_t)$ and $\bar{\lambda}^*(z_t)$ - with $\Sigma_i\lambda^{i*}(z_t)N^{yi}_{t-1} + \bar{\lambda}^*(z_t)z_tN^{yL}_{t-1} = 1$- are the shares of each type of middle age individual in aggregate investment, then the SCMA is achieved. Note how public policy operates here: the pension system implements the efficient investment in physical and human capital by "crowding-out" private saving through social security contributions.

We can interpret the efficient pension system as one with two components, which are described below.

$$\begin{aligned} b^i(z_t) &= \underbrace{t^{ei}(z_{t-1})R^*(z_t)}_{\hat{b}^i(z_t)} + \underbrace{\left(A^{mi*}_{t-1}(z_t) - \tilde{A}^{mi*}(z_{t-1})R^*(z_t)\right)}_{\tau^{oi}(z_t)}, \\ t^{pi}(z_t) &= \underbrace{d^{i*}(z_{t-1})R^*(z_t)}_{\hat{t}^{pi}(z_t)} - \underbrace{\left(A^{yi*}_{t-1}(z_t) + d^{i*}(z_{t-1})R^*(z_t)\right)}_{\tau^{mi}(z_t)}. \end{aligned}$$

For an immigrant we have $\widehat{\bar{b}}(z_t) = \bar{t}^e(z_{t-1})R^*(z_t)$, $\widehat{\bar{t}}^p(z_t) = \widehat{\bar{\tau}}^m(z_t) = 0$ and $\bar{\tau}^o(z_t) = (\bar{A}^{m*}_{t-1}(z_t) - \widetilde{\bar{A}}^{m*}(z_{t-1})R^*(z_t))$.

The first component $(\hat{b}^i(z_t), \hat{t}^{pi}(z_t))$ is used to repay the capitalized value of the educational debt to the lender. The second component $(\tau^{oi}(z_t), \tau^{mi}(z_t))$ is an insurance contract through which the native middle-age and old generations share the immigration risk. The signs of $\tau^{mi}(z_t)$ and $\tau^{oi}(z_t)$ depend on the realization of the shock: when immigration is positive, $\tau^{oi}(\bar{z}) < 0$ for $i = H, L$ and

$\Sigma_i \tau^{mi}(\bar{z}) N_{t-1}^{yi} > 0$, reflecting a transfer from retirees to workers; the opposite in the other case. Also, depending on technological parameters values, the native high human capital workers may or may not have to transfer something to the native low human capital workers: this will depend on how large their income gains are relative to the aggregate increase in output.

Consider the *example of Appendix A*. In the SCMA the return on the saving of the middle age generation in period t is $E_t\left\{\varphi_{t+1}(z)\right\}/p(z_{t+1})$. In an economy without insurance markets the return on the saving is $\varphi(z_{t+1})/p(z_{t+1})$, where $\varphi(z_{t+1}) = p(z_{t+1}) R(z_{t+1})$. To implement the SCMA we have to pick

$$\tau^{oi}(z_{t+1}) = \tilde{A}^{mi*}(z_t)\left[E_t\left\{p_{t+1}^*(z) R_{t+1}^*(z)\right\}/p^*(z_{t+1}) - R^*(z_{t+1})\right]$$

and $\bar{\tau}^o(z_{t+1}) = \tilde{\tilde{A}}^{m*}(z_t)\left[E_t\left\{p_{t+1}^*(z) R_{t+1}^*(z)\right\}/p^*(z_{t+1}) - R^*(z_{t+1})\right]$.

Also, in the SCMA the net income during middle age is equal to $E_{t-1}\left\{p_t(z)\omega_t^i(z)h_t^{mi} - d_{t-1}^i\varphi_t(z)\right\}/p(z_t)$. In an economy without insurance the net income during middle age is equal to $\omega^i(z_t)h_t^{mi} - d_{t-1}^i R(z_t)$. Therefore, to implement the SCMA we have to pick

$$\tau^{mi}(z_t) = \frac{E_{t-1}\left\{p_t^*(z)\omega_t^{*i}(z)h_t^{*mi} - d_{t-1}^{*i}p_t^*(z)R_t^*(z)\right\}}{p^*(z_t)} - \left(\omega^{*i}(z_t)h_t^{*mi} - d_{t-1}^{*i}R^*(z_t)\right).$$

Consider now the case in which, instead of financing education via taxation, the government issues one-period, ear-marked debt in the amount $\Sigma_{i=H,L} d^{i*}(z_t) N_t^{yi}$ in each period. In the following period, the government pays back $\Sigma_{i=H,L} d^{i*}(z_t) R(z_{t+1}) N_t^{yi} + \Sigma_{i=H,L}\tau^{oi}(z_{t+1}) N_{t-1}^{yi} + \bar{\tau}^o(z_{t+1}) z_t N_{t-1}^{yL}$ to the debt holders (where $\tau^{oi}(z_{t+1}) = A_t^{mi*}(z_{t+1}) - \tilde{A}^{mi*}(z_t) R^*(z_{t+1})$ and $\bar{\tau}^o(z_{t+1}) = \bar{A}_t^{m*}(z_{t+1}) - \tilde{\tilde{A}}^{m*}(z_t) R^*(z_{t+1})$). Also in this case, the repayment is financed by a tax on the middle-age individuals that is also computed by adding up two components. The first is proportional to the previous use of public education financing. The second component, again, is for intergenerational insurance. Notice that the net present value of this tax is effectively lump sum for the middle age worker, as it depends only on actions taken when young and on the realization of an exogenous state of the world. In particular, it is not affected by individual's labor supply decisions. In this scheme the government effectively acts as a (somewhat special) financial institution, issuing the missing securities and using its taxing power to enforce repayment that are, efficiently, state contingent.

4.2. Missing insurance markets

The previous analysis shows that in case 1 of section 3, i.e. when agents have access to credit markets to finance education but insurance is not being offered, a PAYGO pension system that always transfers resources from workers to retirees is not efficient. In the absence of private insurance markets, we need a system of intergenerational taxes-transfers contingent on the realization of the immigration shock. Call it $\tau^{mi}(z_t), \tau^{oi}(z_t), \bar{\tau}^{o}(z_t)$ for $i = H, L$. The balance budget of this system reads

$$\sum_{i=H,L} \tau^{mi}(z_t) N^{yi}_{t-1} + \sum_{i=H,L} \tau^{oi}(z_t) N^{yi}_{t-2} + \bar{\tau}^{o}(z_t) z_{t-1} N^{yL}_{t-2} = 0.$$

The budget constraints for a member $i = L, H$ of the generation born in period $t-1$ become

$$\begin{aligned}
&d^i(z_{t-1}) \leq e^i(z_{t-1}) \\
&c^{mi}(z_t) + s^i(z_t) = \omega^i(z_t) h(d^i(z_{t-1}), h^{yi}_{t-1}) - d^i(z_{t-1}) R(z_t) + \tau^{mi}(z_t) \quad \forall z_t \in Z \\
&c^{oi}(z_{t+1}) = s^i(z_t) R(z_{t+1}) + \tau^{oi}(z_{t+1}) \quad \forall z_{t+1} \in Z
\end{aligned}$$

For an immigrant arriving in period t, the budget constraints read

$$\begin{aligned}
&\bar{c}^m(z_t) + \bar{s}(z_t) = \omega^L(z_t) \bar{h}^m_t \\
&\bar{c}^o(z_{t+1}) = \bar{s}(z_t) R(z_{t+1}) + \bar{\tau}^{o}(z_{t+1}) \quad \forall z_{t+1} \in Z.
\end{aligned}$$

Market clearing is

$$\sum_{i=H,L} s^i(z_t) N^{yi}_{t-1} + \bar{s}(z_t) z_t N^{yL}_{t-1} = K_{t+1} + \sum_{i=H,L} d^i(z_t) N^{yi}_t.$$

To implement the SCMA we must set

$$\begin{aligned}
\tau^{oi}(z_t) &= A^{mi*}_{t-1}(z_t) - \tilde{A}^{mi*}(z_{t-1}) R^*(z_t), \\
\bar{\tau}^{o}(z_t) &= \bar{A}^{m*}_{t-1}(z_t) - \widetilde{\bar{A}}^{m*}(z_{t-1}) R^*(z_t), \quad \text{and} \\
\tau^{mi}(z_t) &= A^{yi*}_{t-1}(z_t) + d^{i*}(z_{t-1}) R^*(z_t).
\end{aligned}$$

5. The Welfare State of an Open Economy.

How should the previous analysis be altered in the case of an open economy? Notice first that - if capital mobility is instantaneous and physical capital (and high human capital) flows into the country at the same speed at which low skill immigrants do so as to restore equality between the internal rate of return on capital and the one established on the international capital markets - the immigration shock has no relevance whatsoever as neither the wage of the native workers nor the return on capital of the native capital owners will be affected by the arrival of new workers. Efficient and frictionless capital and labor markets may act, in this context, as insurance devices rendering the state contingent assets essentially redundant. This is an interesting result as it suggests that, in the light of the simulations presented earlier, the Spanish trade deficit was beneficial, in terms of consumption and overall utility, to both the native households and the immigrant ones.

This observation helps explaining, at least in part, what we observed in Spain from 1996 until the arrival of the global financial crisis in 2007-2008: as the flow of immigration into the country continued and even increased, Spain external trade deficit balooned while productivity did not move. Along the lines of our model, these facts have the following interpretation: capital flew into Spain if not at the same rate at which labor was entering, certainly at a very high rate, thereby preventing the real wage of unskilled worker from falling. Analysis of the actual data is difficult, not to say impossible, by means of a model as simplified as this, in which there is no distinction between durable and non durable goods and one period lasts roughly thirty years of which, since the immigration shock first hit, we have observed at most half. To put it differently: if one takes our model literally, there is no sense in which it can be used to study the Spanish case because we have not yet completely observed even a single "model period" in the actual data.

Nevertheless, a simple back of the envelope calculation based on our model should tell us how much the immigration shock contributed to the Spanish trade deficit of the period 1996-2007. We do know that immigrants probably have a slightly lower human capital level than native, but we do not have good estimates of the ratio so, let us assume for simplicity that workers are homogenous within each generation and immigrants have the same human capital that natives. In our model this implies that all workers have the same skill level and the firms use only K and L to produce. This is a big simplification, but it can provide us with a useful benchmark. Assume, therefore, that capital flew into Spain at roughly the rate needed, year after year, to keep the internal K/L ratio constant.

The bottom line of the calculation is the following. In Spain the annual K/Y ratio was about 2.9 without housing and 4.0 with housing in 1996. Employment in 1996 was still about 12.8 M while it was 20.3 M in 2007, of which 2.7 M were immigrants. The K/Y ratio increased slowly during the expansion reaching about 3.1 in 2007. Hence, with respect to the original work force the immigrants added almost 21%, while they were about 13% of the 2007 work force. Take a number in between (i.e. 17%) to account for the fact that this process took place over, roughly, a decade (a little less, in fact). Because a period in our model is about three times as long as the amount of time considered in the data, everything should be scaled accordingly.

This implies that, if (i) the immigrants came without any K; (ii) the saving rate of the natives remained constant (it roughly did) meaning that, if Spain was in steady state before 1996, national saving just supplied the K needed by the natives; and, (iii) the final K/L ratio for immigrants is similar to those for natives; then Spain would have had to borrow from abroad the resources needed to increase its original stock of K of about 17%. In fact, the number is larger because, on top of the 2.7 M immigrant workers, we have about other 4.0 M native workers that became employed and were not such, at least officially, when the expansion period started. The quantitative problem with the native workers is more complicated as part of them were probably already working in the underground economy (hence, their K already existed), part had accumulated savings they invested in their own K, and so on and so forth. In any case, because the employment growth attributable to natives would only add to our estimated demand for capital to be imported from abroad, the reference value of 17% based only on the immigrants inflow will be a very reasonable lower bound.

Summing up: when applied to the 1996-2007 period our simplified model predicts that, if international capital markets were functioning properly, Spain should have imported an amount of capital equal to, at least, 17% of its initial capital stock. Notice that "imported" here means "net import", as there is no export in our model: import in the model is equal to the trade deficit in the national income accounts. This means that, over a period of about 11 years, Spain had to import a little less than 1/5 of its original stock of capital; that is about 1.5-1.6% of its capital stock per year every year between 1996 and 2007. Because, during those years, the K/Y averaged about 3.0 (4.0 when housing is considered) this implies that, each year, something between a lower boud of 4.5% and an upper bound of 6.4% of GNP had to be imported. That yields a cumulated total import of between 50% and 71% of Spanish GNP, everything else equal. Between 1996 and 2007, the actual trade deficit, in percentage of GNP, adds up to 50.7% with an annual average of 4.2%. Sheer chance? Maybe.

6. Conclusions

We have carried out a straightforward exercise. We built the simplest dynamic model in which immigration shocks have both aggregate and distributional effects, affecting some part of the population differently from other. In particular, in our model a positive immigration shocks increases the labor supply of unskilled middle-age workers making the high skill ones and the (older) owners of capital better off, apart from increasing GNP. Next we asked how a system of (sequentially) complete financial markets would handle such shocks and characterized the properties of the sequentially complete markets allocations (SCMA). Finally, we have asked two kinds of questions. First, if financial markets were not complete, in a practically meaningful sense, what kind of welfare policies could bring back the SCMA? Second, if international capital markets were frictionless, how much capital should a country import to insure against the immigration shock? Our analysis provides us with four interesting morals, which we can apply to the case of Spain (1996-2007).

Moral number 1: immigration shocks have large impacts not only on aggregate output but also on its composition and on income distribution. Absent complete financial markets, such impacts should be properly managed by well planned government policies, of the form we have described. It is at least dubious that such policies were implemented in Spain during the period under consideration. The absence of such policies has clear detrimental effects not only on welfare but also on human capital accumulation and overall economic growth.

Moral number 2: the trade deficit and borrowing from abroad can be a substitute for the missing internal insurance markets. Spain received a very large number of immigrants and this would have had a dramatic impact on productivity and income distribution if the trade deficit had not allowed the country to accumulate capital stock much faster than the national saving rate allowed. This generated a large trade deficit, but increased output, wages, consumption and, overall, welfare.

Moral number 3: the debate on the impact of immigration on the Spanish society and economy seems to be missing some key aspects. In the model presented here we have outlined some of them, with particular focus on education and pensions. In particular, we have shown that an optimal policy response to a large immigration flow requires a reduction of pension payments and an increase of the investment in education. As far as we can tell, neither of these two policies were implemented in Spain between 1996 and 2007. Five years later, they do not seem to be yet on the political agenda.

Moral number 4: even simple stylized models can help thinking about difficult issues in economic policy and are capable of shed new light on issues that are often forgotten or considered too complicated to be addressed formally.

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Appendix A: Analytical illustration

In this analytical example we consider an economy with logarithmic utility and Cobb Douglas production functions: $u(c) = \log c$, $F(K, H, L) = AK^{\alpha}H^{\theta}L^{1-\alpha-\theta}$ and $h\left(d^i, h^{yi}\right) = B(d^i)^{\beta}\left(h^{yi}\right)^{1-\beta}$.

Write $W^i(z_t) = \omega^i(z_t)h(d^i_{t-1}, h^{yi}_{t-1}) + A^{yi}_{t-1}(z_t)$ -the net income of individual i in period t and state z_t-. From (3.c) we have, for a native middle age of type i,

$$q(z, z_t) = \frac{\pi(z|z_t)\delta}{A^{mi}_t(z)}\left[W^i(z_t) - \widetilde{A}^{mi}(z_t)\right] \ \forall z \in Z,$$

where $\widetilde{A}^{mi}(z_t) = \sum_{z\in Z} q(z, z_t)A^{mi}_t(z)$. Multiplying by $A^{mi}_t(z)$ and aggregating in $z \in Z$ we arrive at the total demand for contingent securities of native middle age individuals of either type,

$$\widetilde{A}^{mi}(z_t) = \frac{\delta}{1+\delta}W^i(z_t).$$

The demand for consumption in middle-age, and the demand for each component $A^{mi}_t(z)$ of $\mathcal{A}^{mi}(z_t)$, are

$$c^{mi}(z_t) = \frac{1}{1+\delta}W^i(z_t) \qquad \text{and} \qquad A^{mi}_t(z) = \frac{\delta}{1+\delta}W^i(z_t)\frac{\pi(z|z_t)}{q(z, z_t)}.$$

Write $\bar{W}^i(z_t) = \omega^L(z_t)h^{mL}_t$. For an immigrant in t we get

$$\widetilde{\bar{A}}^{m}(z_t) = \frac{\delta}{1+\delta}\bar{W}^i(z_t), \ \ \bar{c}^m(z_t) = \frac{1}{1+\delta}\bar{W}^i(z_t) \ \text{ and } \ \bar{A}^m_t(z) = \frac{\delta}{1+\delta}\bar{W}^i(z_t)\frac{\pi(z|z_t)}{q(z, z_t)}.$$

Using condition (3.b) and the first order conditions for the firm we have

$$d^H(z_{t-1})\, N^{yH}_{t-1} = \frac{\beta\theta}{\alpha}K_t \quad \text{and} \quad d^L(z_{t-1})\, N^{yL}_{t-1} = \frac{\beta(1-\alpha-\theta)}{\alpha}\Psi(z_{t-1})\, K_t,$$

where $\Psi(z_{t-1}) = E_{t-1}\{p_t(z)\,(1+z)^{-\alpha-\theta}\}/E_{t-1}\{p_t(z)\,(1+z)^{1-\alpha-\theta}\}$. From (3.$a$) we have $p_t(\bar{z})\, c^{mi}_t(\bar{z}) = p_t(0)\, c^{mi}_t(0)$, for $i = H, L$. Then, using the condition for the firm (1.c) and the consumer budget restriction (2.c) we get the demand for each component $A^{yi}_{t-1}(z)$ of $\mathcal{A}^{yi}(z)$:

$$A^{yi}_{t-1}(z) = \frac{E_{t-1}\left\{p_t(z)\,\omega^i_t(z)h(d^i_{t-1}, h^{yi}_{t-1}) - d^i_{t-1}\varphi_t(z)\right\}}{p_t(z)} - \omega^i_t(z)h(d^i_{t-1}, h^{yi}_{t-1}).$$

It is important to note that in the SCMA the net income during middle age in equilibrium is equal to $E_{t-1}\left\{p_t(z)\,\omega_t^i(z)h(d_{t-1}^i,h_{t-1}^{yi})-d_{t-1}^i\varphi_t(z)\right\}/p(z_t)$. Now, using (6.$c$)-(6.$d$) and taking into account that $A_t^{mi}(z)=\widetilde{A}^{mi}(z_t)\pi(z|z_t)/q(z,z_t)$ and $\bar{A}_t^m(z)=\widetilde{\bar{A}}^m(z_t)\pi(z|z_t)/q(z,z_t)$, we have the aggregate demand for each component $A_{t-1}^y(z)$ of $\mathcal{A}^y(z)$:

$$\sum_{i=H,L}A_{t-1}^{yi}(z)N_{t-1}^{yi}=-D\left(z_{t-1}\right)\frac{\pi(z|z_{t-1})}{q\left(z,z_{t-1}\right)}+K_t\left[\frac{\varphi_t\left(z\right)}{p_t\left(z\right)}-\frac{\pi(z|z_{t-1})}{q\left(z,z_{t-1}\right)}\right].$$

Finally, from (1) we obtain the equilibrium prices for each period t and state $z\in Z$:

$$\begin{aligned}
\omega_t^H(z) &= \theta AK_t^\alpha H_t{}^{\theta-1}L_t\left(z\right)^{1-\alpha-\theta}\\
\omega_t^L(z) &= \left(1-\alpha-\theta\right)AK_t^\alpha H_t{}^{\theta}L_t\left(z\right)^{-\alpha-\theta}\\
\varphi_t(z) &= p_t\left(z\right)\alpha AK_t^{\alpha-1}H_t{}^{\theta}L_t\left(z\right)^{1-\alpha-\theta}\\
q(z,z_t) &= \frac{\pi(z|z_t)p_{t+1}\left(z\right)}{E_t\left\{p_{t+1}\left(z\right)\alpha AK_{t+1}^{\alpha-1}H_{t+1}{}^{\theta}L_{t+1}\left(z\right)^{1-\alpha-\theta}\right\}}.
\end{aligned}$$

Note also that, in equilibrium, $p_t(\bar{z})c_t^{mi}(\bar{z})=p_t(0)c_t^{mi}(0)$. Substituting the values of $c_t^{mi}(\bar{z})$ and $c_t^{mi}(0)$ we arrive to $p_t\left(\bar{z}\right)=p_t\left(0\right)\left(1+\bar{z}\right)^{\alpha+\theta}/\left(1+\left(\alpha+\theta\right)\bar{z}\right)$, where $\left(1+\bar{z}\right)^{\alpha+\theta}<1+\left(\alpha+\theta\right)\bar{z}$ and we normalize $p_t\left(0\right)=1$.

Given initial conditions for $\left(K_0,H_0,L_0,N_0^{oi},N_0^{mi},N_0^{yi},A_{-1}^{yi}\left(z_0\right),A_{-1}^{mi}\left(z_0\right),A_{-1}^{f}\left(z_0\right)\right)$, the following system describes the dynamic of the economy for a given sequences of shocks $(z_0,z_1,...)$ and a sequence of endowment of basic knowledges $\left\{h_t^{yH},h_t^{yL}\right\}_{t=0}^{\infty}=\left\{H_t/N_t^{yH},L_t/N_t^{yL}\right\}_{t=0}^{\infty}$,

$$K_{t+1} = \Omega\left(z_t,z_{t-1}\right)AK_t^\alpha H_t^\theta L_t^{1-\alpha-\theta}, \tag{A.1}$$

$$H_{t+1} = B\left[\frac{\beta\theta\Omega\left(z_t,z_{t-1}\right)}{\alpha}\right]^\beta A^\beta K_t^{\alpha\beta}H_t^{\theta\beta+1-\beta}L_t^{(1-\alpha-\theta)\beta}, \tag{A.2}$$

$$L\left(z_{t+1}\right) = \left(1+z_{t+1}\right)\left[\frac{\beta(1-\alpha-\theta)\Psi\left(z_t\right)\Omega\left(z_t,z_{t-1}\right)}{\alpha}\right]^\beta A^\beta K_t^{\alpha\beta}H_t^{\theta\beta}L_t^{1-\alpha\beta-\theta\beta}, \tag{A.3}$$

where

$$\Omega\left(z_t, z_{t-1}\right) = \frac{\delta}{(1+\delta)\,\Theta\left(z_t\right)}\left[1-\frac{\left(1+(\alpha+\theta)\,z_t\right)\alpha\Theta\left(z_{t-1}\right)}{\left(1+z_t\right)}\left[\pi(0)+\frac{\pi(\bar{z})\,(1+\bar{z})}{(1+(\alpha+\theta)\,\bar{z})}\right]\right],$$

$$\Theta\left(z_{t-1}\right) = 1+\frac{\beta}{\alpha}\left(\theta+(1-\alpha-\theta)\Psi\left(z_{t-1}\right)\right),$$

and

$$\Psi\left(z_{t-1}\right) = \frac{\pi(0|z_{t-1})+\pi(\bar{z}|z_{t-1})\frac{1}{1+(\alpha+\theta)\bar{z}}}{\pi(0|z_{t-1})+\pi(\bar{z}|z_{t-1})\frac{(1+\bar{z})}{1+(\alpha+\theta)\bar{z}}}.$$

Given a sequence of shocks $(z_0, z_1, ..)$ the evolution of the factor intensity ratios $\tilde{k}=K/L$ and $\tilde{h}=H/L$ are given by

$$\tilde{k}\left(z_{t+1}\right) = \frac{\left(A\Omega\left(z_t, z_{t-1}\right)\right)^{1-\beta}}{\left(1+z_{t+1}\right)B\left(\frac{\beta(1-\alpha-\theta)\Psi(z_t)}{\alpha}\right)^{\beta}}\tilde{k}_t^{\alpha(1-\beta)}\tilde{h}_t^{\theta(1-\beta)}$$

$$\tilde{h}\left(z_{t+1}\right) = \frac{1}{\left(1+z_{t+1}\right)}\left(\frac{\theta}{(1-\alpha-\theta)\Psi\left(z_t\right)}\right)^{\beta}\tilde{h}_t^{(1-\beta)}$$

Set $(z_t, z_{t+1}, z_{t+2}, ...) = (0,0,0,...)$. The rays

$$\tilde{h}^* = \frac{\theta}{(1-\alpha-\theta)\Psi\left(0\right)}, \qquad \text{and}$$

$$\tilde{k}^* = \left[\frac{\left(\Omega\left(0,0\right)\right)^{1-\beta}}{B\left(\frac{\beta(1-\alpha-\theta)\Psi(0)}{\alpha}\right)^{\beta}}\left(\frac{\theta}{(1-\alpha-\theta)\Psi\left(0\right)}\right)^{\theta(1-\beta)}\right]^{\frac{1}{1-\alpha(1-\beta)}}$$

define a balanced growth path. For all initial conditions $(H_0, K_0, L_0) \in \Re^2_+$, iteration of $(A.1)-(A.3)$ leads (H_t, K_t, L_t) to the rays $\tilde{h}^*$ and $\tilde{k}^*$.

Along the balanced growth path, the three stocks of capital expand (or contract) at the factor

$$1+g^* = \Omega\left(0,0\right)A\left[\frac{\left(\Omega\left(0,0\right)\right)^{1-\beta}}{B\left(\frac{\beta(1-\alpha-\theta)\Psi(0)}{\alpha}\right)^{\beta}}\right]^{\frac{\alpha-1}{1-\alpha(1-\beta)}}\left(\frac{\theta}{(1-\alpha-\theta)\Psi\left(0\right)}\right)^{\frac{\theta\beta}{1-\alpha(1-\beta)}}.$$

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