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The arterial pattern and fractal dimension of the dog kidney

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Summary. A method has been developed by which it is possible to measure the fractal dimension of the arterial tree of the kidney. The objective of this work is to determine a method which permits us to discriminate between the architectures of specific organs by reference to a unique number, namely the fractal dimension of the arterial tree of that organ. This method opens the possibility of a new taxonomy for normal organs and for the pathological injuries related to the vascular morphology of those organs.

The method that we have devised uses as its input the volume which is taken up by the arterial tree of the kidney. In order to calculate this volume we first obtained a plastic cast (the arteries were filled with Araldite CY233 plastic resin after which the organic tissues were corroded); thereafter we constructed a theoretical arterial tree having the same volume as the renal one. From this simplified tree, we were able to calculate its fractal dimension.

The complete process of constructing the theoretical arterial tree and the subsequent calculation of its fractal dimension was carried out automatically by way of a computer programme to which we have given the name fractal program.

Key words: Fractal, Arterial, Kidney, Dog

Introduction

Classically, the morphological sciences, when studying an organ, describe it by reference to its macroscopic and microscopic anatomy. Some of the features that define the organ are qualitative and provide subjective data; others are quantitative, but they give rise to values which are not discriminatory.

The possibility of being able to characterise an organ by means of a single, unique and objective value, for example a number, is, we suggest, an attractive idea, but it presents us with a fundamental problem, namely that of choosing the method which will allow us to obtain that unique number for each organ. Could that method be its fractal dimension?

Kidney anatomy of the arterial tree

Fuller and Huelke (1973) have said «the renal arteries in the dog arise from the aorta, slightly caudal to the celiac and superior mesenteric trunks. The renal arteries divide into dorsal and ventral rami well before entering the hilum of the kidney. The ventral rami of both renal arteries pass toward the hilum dividing into two stem branches, each of which may again divide before reaching the hilum of the kidney. Within the substance of the kidney, these ventral branches supply the cranial, middle and caudal areas of the ventral surface of the kidney, and are named according to their distribution (ventral-cranial, ventral-middle, and ventral-caudal segmental arteries). Usually each of these segmental arteries is double. The dorsal ramus of the renal artery likewise divides into double branches for the supply of the cranial, middle and caudal portions of the dorsal half of the kidney. The caudal third of the dog kidney is always supplied by branches from both the dorsal and ventral rami». (Nickel et al., 1973).

What is a fractal?

In classical Euclidean geometry objects have 0, 1, 2 or 3 dimensions. Modern algebra makes it possible for us to add more dimensions to classical geometry, 4, 5, 6... etc, and this is useful when solving certain problems with physics. Through the use of algebra, man has been able to both imagine and manage abstract objects having

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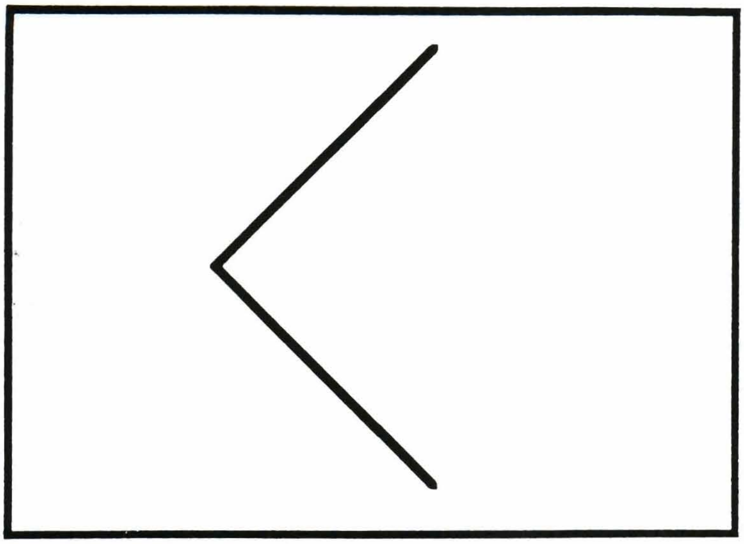


Fig. 1. The *V*-shaped fractal pattern used in this work.

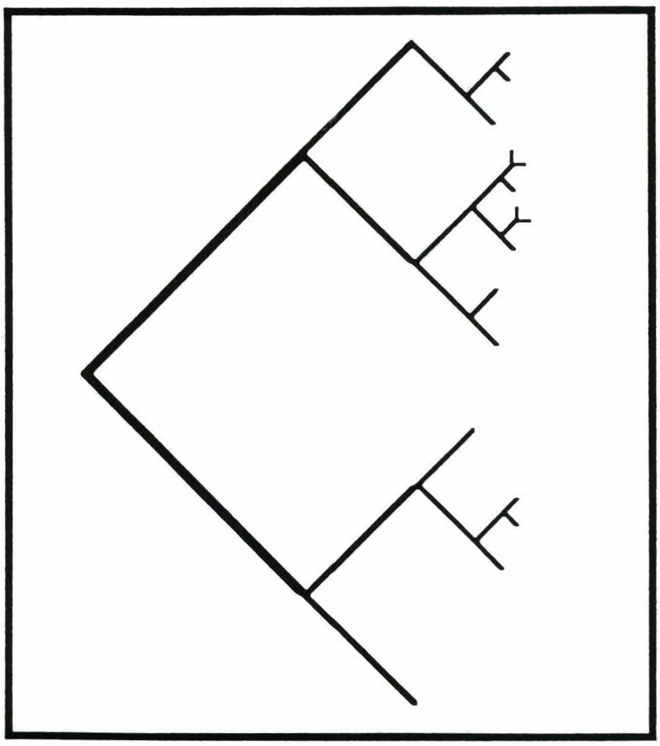


Fig. 2. Section of a theoretical arterial tree.

more than three dimensions, although paradoxically he has not been able to draw many of them. Both classical geometry and its subsequent extensions always require an integer number of dimensions (1, 2, ...N).

But there is another geometry which uses fractional (1/2, 3/2, etc), or real dimensions (log 4/log 3, etc). Those objects to which we can assign a fractional or real dimension are called *fractal objects* and the value corresponding to the fractional dimension is called the *fractal dimension* (Mandelbrot, 1987) or Hausdorff-Besicovitch dimension (Abbot and Wise, 1981). However nobody has yet defined a fractal objectively.

We are able to recognize a typical fractal object by its irregular and/or fragmented appearance; its complexity, however, is only apparent, and this is because its geometry has been constructed by scaling a single geometry entity (a straight line, a curve, etc) several times. The property of scaling is expressed mathematically by saying that it has interior similarity or self similarity.

Fractals in biology

A number of pathophysiological cardiac disturbances display fractal behaviour (Goldberger and West, 1987).

Fractal morphology can be attributed to trees, feathers, networks of neurons, His-Purkinje system, vessel trees and other networks (Meakin, 1986; Goldberger and West, 1987; Barnsley, 1988; Goldberger et al., 1990). In Biology the most intensively studied fractal morphology is that of the bronchial tree of the human lung (Weibel and Gómez, 1962; Goldberger and West, 1987).

In order to calculate the fractal dimension of an organ, the method which is habitually employed requires a large number of measurements of that organ. These measurements can be length, surface areas, or volumes of successive segments of a tree. They are plotted against the generation or branch numbers of the segments on log-log graphs. If all points fall on a straight line, the negative of its slope is defined as the fractal dimension (Tsonis and Tsonis, 1987).

This method is so laborious that the fractal dimensions of many other organs are still to be quantified or, if this process has been carried out, then it has been with little precision, so that the arterial and venous cast of a kidney has a fractal dimension of between 2 to 3 (Sernetz et al., 1985).

Our work on the fractal dimension

In this work we describe a method which we have ourselves developed to calculate the fractal dimension of the arterial tree of the dog's kidney, a method which is characterized by the need to take very few measurements. This method can be applied to any organ in any species, whenever it is possible to obtain just a few measurements of the arterial tree.

On the basis of the results obtained by using our method we discuss the question which we have earlier posed: namely, can the fractal dimension be a single unique number which serves to define an organ in a species?

Materials and methods

Preparation of the casts

For the purpose of our study we have used three crossbred male adult dogs weighing between 25-35 kg, provided by the Service of Animal Protection. The animals were first anaesthetized with Combelen (1.5 cm^3/animal, by way of intramuscular administration) and with 0.5 gm/animal of pentobarbital and 500 U.I./kg live weight sodium heparin, both administered intravenously; the animals were then sacrificed with an overdose of pentobarbital (1.5 gm/animal, by way of intravenous administration).

The left jugular vein was dissected and a glass canula inserted orientated towards the heart. The vascular system was perfused with a cleansing solution made up of 6 to 8 litres of phosphate buffer, 0.1 M, pH 7.3 and was drained by the same jugular vein through the craneal limit of the incision we had earlier made. Once the vascular system had been washed, it was filled with 2 littres of 2% glutaraldehyde solution adjusted to pH 7.3 with the addition of 1% of glucose. The washing solution as well as the fixing solution were administered at 39° C.

After the abdominal cavity was opened, the kidneys were extracted in order for them

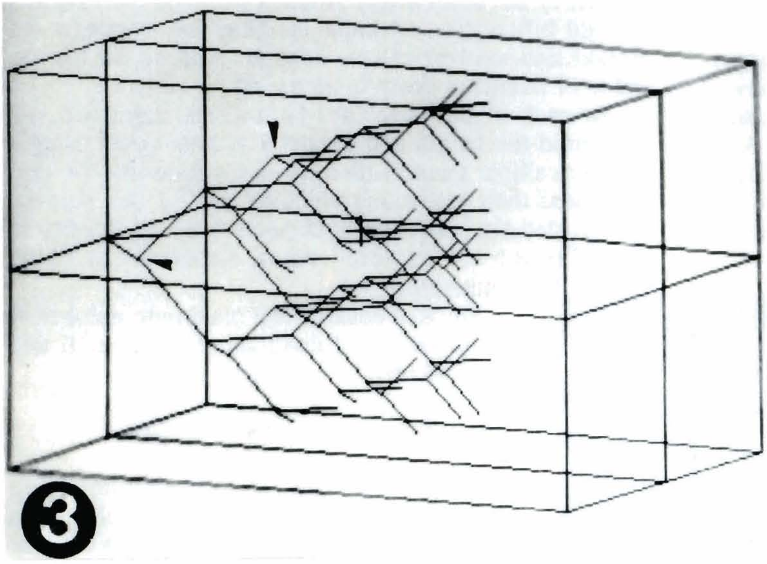


Fig. 3. Portion of one theoretical arterial tree. Dichotomic bifurcations (arrows). A whole kidney is represented by means of a prism.

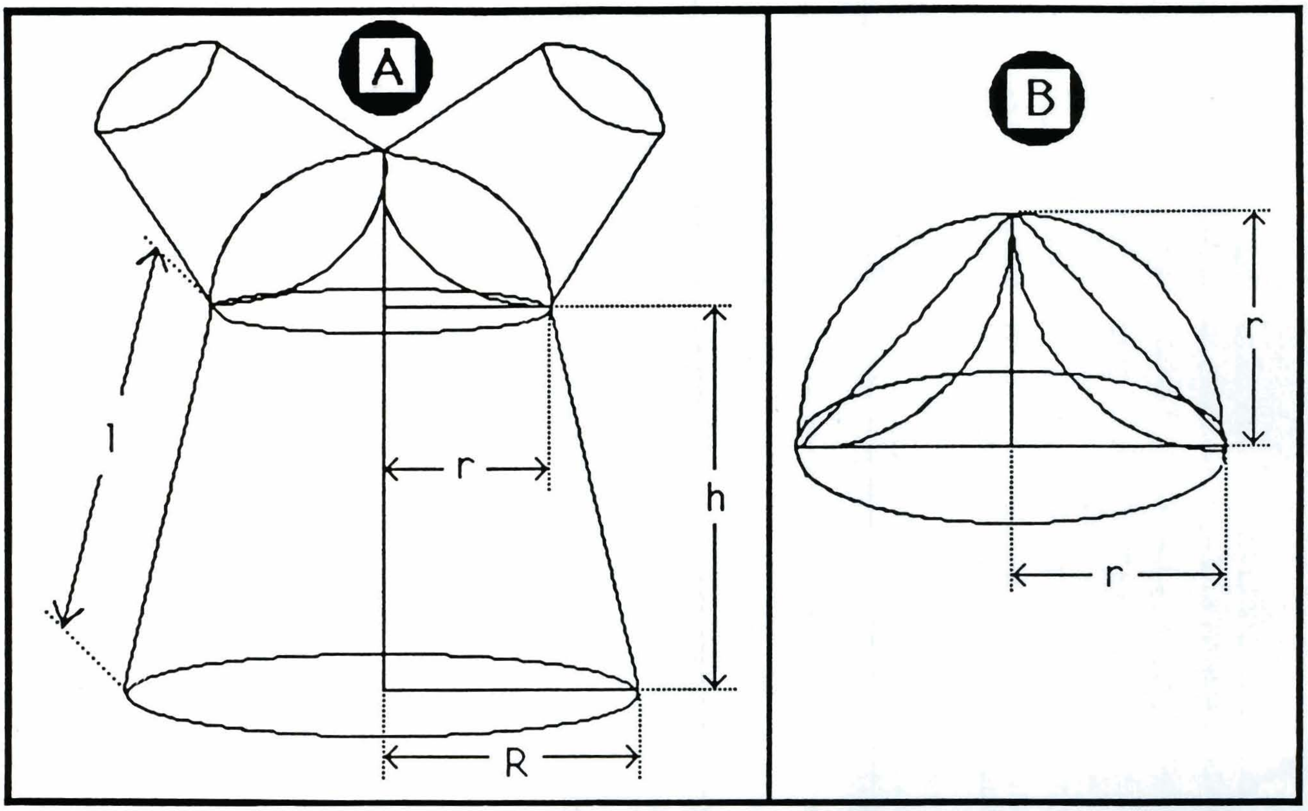
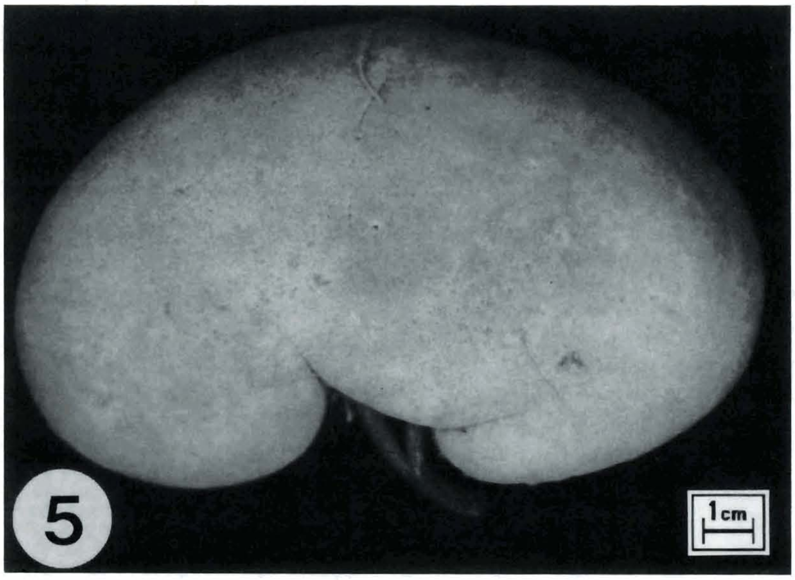


Fig. 4. Volumes that surround each branch of the *V*-shaped forks. **A:** junctions of the successive bifurcations; R is the radius of the largest base of the truncated cone, r is that of the smallest base, h is the height which measures the same as an arm, or branch, of the *V*-shaped fractal. **B:** geometric body of the junctions in the bifurcations with r representing both the height and the radius of the largest base.

to be weighed and their volume measurement taken. The respective renal arteries were tubed with a 14 gm Avocatto cannula and a mixture of epoxidic Araldite CY223 resin and polyaminic HY 2967 (Ciba-Geigy) hardener was injected in a volume proportion 87 cm^3/40 cm^3. The resin had itself been previously mixed with 2% DW 0133 red colourant. All the products had been kept in a bath at 40° C up to the moment of injection.

Following the injection of the resin, the organ was kept for two hours at room temperature, a sufficient time for the polymerization of the products. The kidneys were submerged in a 20% KOH solution and were kept for 24 hours in a stove at 50° C in order for tissue corrosion to take place and for the extraction of the plastic cast, which was then throughly cleaned in running water.



Measurements taken from the plastic casts

Measurements were taken at 13 sites on each of the plastic casts: the first site chosen was the cast of the renal artery, from its separation from the abdominal aorta artery to its first bifurcation into dorsal and ventral rami *(*Branch 0*)*; the second was the cast of the main branch of the renal artery situated between the first and second bifucartion (Branch 1). Then we averaged out dorsal and ventral measurements. And so on for the other 11 Branches down to glomeruli.

At each of the four sites (Branches) described, we measured the length and diameter at both bases using a vernier calliper gauge with a 0.1 mm sensitivity. We also measured their respective volumes and for this purpose we divided the weight of each section by the density of the plastic: 1.1 gm/cm^3 when hardened at room temperature.

We also counted the maximum number of dichotomous bifurcations of the arterial tree that appeared on each plastic cast.

Calculation of the fractal dimension

We set out with the aim of simplifying and automating the mathematical calculation process. We have achieved simplicity in that our method needs very few measurements for input: for the plastic cast as a whole, only the measurement of its volume and the maximum number of bifurcations that it posses and the volume, length and the radius at each base of one of its Branches (0 to 12) are used. With respect to automatization, we constructed a fractal programme in the form of a computer program written in Pascal Light Speed 2.0 and executed on a Macintosh Plus computer (more details can be found in the attached Appendix).

The programme has been designed to carry out various tasks. First it is able to construct theoretical trees that are similar to the skeletons of the trees which can be observed in each plastic cast. Note that we have not attempted to construct an exact template of each plastic cast. Each theoretical tree is formed by repeating, at different scales, the fractal pieces in the form of a *V*, to which we have given the name fractal model (Figs. 1-3). As the theoretical tree grows, the programme by mean one fractal dimension relating these successive pieces. The *V* shaped fractal dimension is different for each tree constructed by the programme.

Fig. 5. Whole cast, dorsal view.

Fig. 6 Whole cast, section. One dichotomic bifurcation (arrow).

Secondly, the programme calculates the total volume of each theoretical fractal tree by totalling the volumes of all the truncated cones constructed around that tree; for this the program used each *V* Branch of each theoretical fractal tree as the axis of a skeleton from which we are able to construct a truncated cone (Fig. 4).

Finally, from amongst the theoretical trees which have been constructed, the programme selects those which have the same volume as the plastic cast ± 0.05%.

Each tree constructed by the programme has a maximum of 10^{12} Branches because this number is the maximum number of dichotomic bifurcations that we noted. For each set of data 100 different theoretical trees are constructed, the fractal dimensions of which are differentiated by a value equal to 0.01. The programme carries out all calculations with 8-digit real numbers.

The fractal dimension connected with each *V*-shape which forms the theoretical tree selected by the programme is the dimension allocated to the actual kidney by this fractal programme.

Results

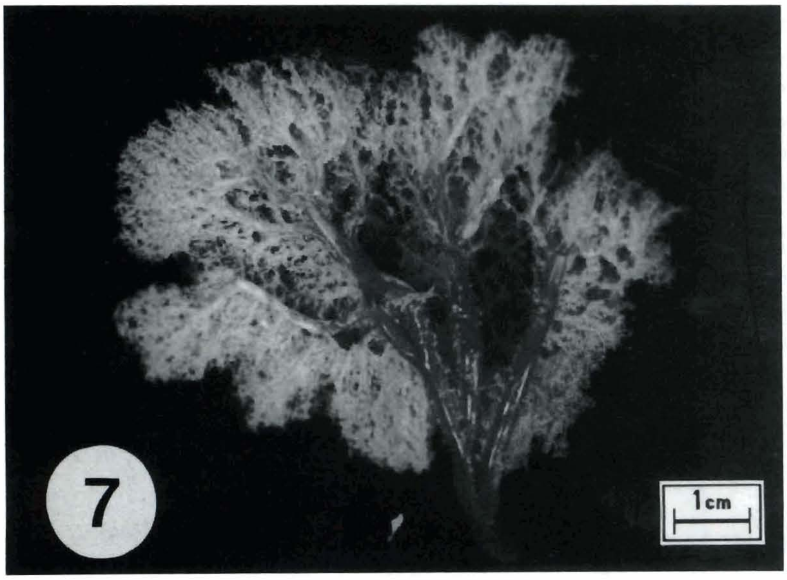
The epoxidic resins enabled us to obtain complete casts of the kidney (Figs. 5, 6). Whilst the arterial system was filled homogeneously, we noted that the vascular system was not filled from the efferent arterioles onwards (Figs. 7, 8), but the filling did reach the straight vessels that come from the arch-shaped arteries.

With the aid of a magnifying glass we noted that the maximum number of dichotomic bifurcations was 12, located from the first division of the interlobed arteries to the glomeruli located in the cortical, which meant the existence of up to 10^{12} Branches in each plastic cast.

The measurements obtained from these plastic casts of the arterial trees from their separation from the abdominal aorta artery down to glomeruli, are shown in Table 1, and these same numerical values were introduced into the computer programme as its input. The volume of both the efferent arterioles (Fig. 8) and of the glomeruli were excluded in the volume of the plastic casts of three kidneys; they were, respectively 4.22%, 9.61% and 7.56% (Table 1B). The theoretical volumes were calculated down to the glomerulus (Tables 2, 3).

After carrying out the calculations, the fractal programme detected several fractal dimensions for which there was a coincidence when comparing the volume of their theoretical fractal trees and the volumes of their respective plastic casts. When comparing volumes we allowed for an error of ± 0.05%. We noted that only the fractal dimension having a value of 1.94 was found in all the results, whether the fractal programme employed as its input data the set of measurements relating to the first Branch, or whether it employed those relating to the second.

Tables 2 and 3 show the measurements for the branches of the theoretical fractal trees constructed for each of the three kidneys when the fractal dimension is 1.94 and when we employed as input the data from Branches 1 or 2, as taken from their respective plastic casts. We also calculated the correlations (Table 4) of the measurements of these theoretical fractal trees (Tables 2, 3) with those corresponding



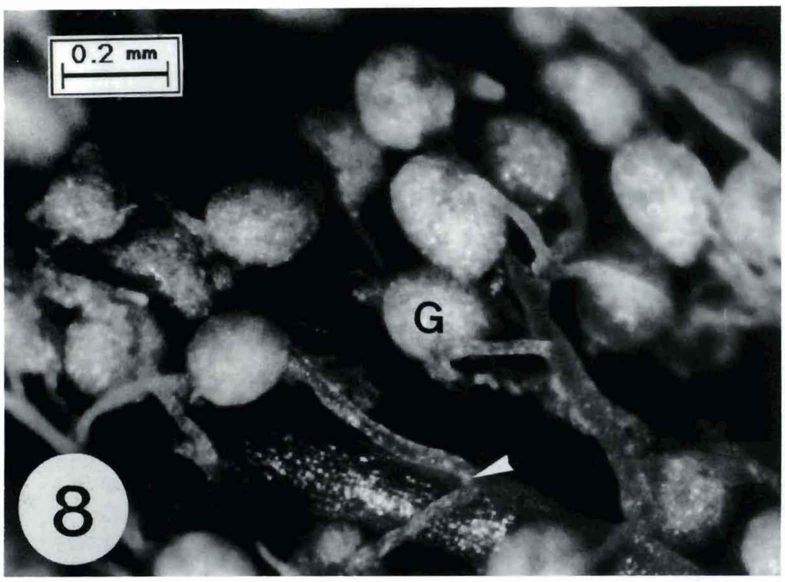


Fig. 7. Cast of efferent arterioles.

Fig. 8. Cast of efferent arterioles and their glomeruli (G). One dichotomic bifurcation (arrow).

Table 1. Set of measurement obtained from the plastic casts and used by the programme successively as input. Branch number 0 is renal artey, from its separation from the abdominal aorta artery to its first bifurcation into dorsal and ventral rami. C is the volume of both the efferent arterioles and of the glomeruli.

Kidney identification number		1		2		3
Total volume of kidney (mm^3)		73000		80000		70000
(A) Total volume of plastic cast		6222		6149.72		5256.8
Maximun number of bifurcations from the renal artery to the glomerulus		12		12		12
Kidney number	Branch number	Length (mm)	Volume (mm^3)	Diameter of largest base (mm)	Diameter of smallest base (mm)	Total volume of all the branches equal to this (mm^3)
1	0	12.45	141.00	4.30	4.05	141.000
1	1	14.00	47.09	4.00	3.00	94.180
1	2	11.00	15.18	2.50	2.40	60.720
1	3	10.00	14.91	2.40	2.05	119.272
1	4	1.08	0.91	2.05	0.28	14.555
1	5	0.90	0.81	0.28	0.18	25.821
1	6	0.76	0.75	0.18	0.13	48.282
1	7	0.65	0.72	0.13	0.09	92.658
1	8	0.56	0.70	0.09	0.07	180.264
1	9	0.49	0.69	0.07	0.05	354.047
1	10	0.43	0.68	0.05	0.04	698.651
1	11	0.39	0.68	0.04	0.04	1383.455
1	12	0.35	0.67	0.04	0.03	2745.993
2	0	14.40	79.63	4.50	4.20	79.630
2	1	10.00	40.18	3.50	3.20	80.360
2	2	8.00	17.09	2.90	2.70	68.360
2	3	9.00	13.54	2.50	2.10	108.320
2	4	4.12	1.66	2.10	0.90	26.56
2	5	2.73	1.50	0.90	0.54	47.928
2	6	1.84	1.21	0.54	0.34	77.336
2	7	1.26	1.01	0.34	0.21	129.738
2	8	0.87	0.87	0.21	0.13	223.777
2	9	0.60	0.77	0.13	0.08	394.060
2	10	0.42	0.69	0.08	0.05	705.097
2	11	0.29	0.62	0.05	0.03	1277.719
2	12	0.21	0.57	0.03	0.02	2339.253
3	0	9.30	88.09	4.00	3.25	88.090
3	1	10.00	11.36	2.00	1.90	22.720
3	2	7.00	5.18	1.30	1.10	20.720
3	3	6.00	1.91	1.00	0.90	15.272
3	4	1.12	0.26	0.90	0.86	4.161
3	5	0.86	0.76	0.86	0.56	24.462
3	6	0.68	0.69	0.56	0.37	44.053
3	7	0.56	0.64	0.37	0.24	82.468
3	8	0.47	0.62	0.24	0.16	157.777
3	9	0.41	0.60	0.16	0.11	305.854
3	10	0.35	0.58	0.11	0.07	597.915
3	11	0.31	0.57	0.07	0.05	1175.469
3	12	0.28	0.57	0.05	0.03	2319.975
(B) On each kidney addition of total volume of all branches				5958.99	5558.137	4858.93
(C)= A-B				263.01	591.58	397.87

Table 2. Measurements pertaining to the branches of the theoretical fractal trees of the three kidneys, when the fractal dimension is 1.94 and when, for input, the data from Branch 1 (the main branch of the renal artery situated between the first and second bifurcation) of their respective plastic casts is employed. Branch number 0 is renal artery, from its separation from the abdominal aorta artery to its first bifurcation into dorsal and ventral rami.

Kidney identification number			1	2		3
Maximun number of bifurcations from the renal artery to the glomerulus			12	12		12
Kidney number	Branch number	Length (mm)	Volume (mm^3)	Diameter of largest base (mm)	Diameter of smallest base (mm)	Total volume of all the branches equal to this (mm^3)
1	0	12.45	41.0000	4.300	4.050	141.000
1	1	14.00	47.0900	4.000	3.000	94.180
1	2	4.38	9.6780	3.000	2.687	38.712
1	3	27.82	419.39	2.687	1.630	3355.152
1	4	15.56	86.46	1.630	0.989	1383.468
1	5	8.719	17.84	0.989	0.600	571.096
1	6	4.87	3.67	0.600	0.364	235.383
1	7	2.73	0.7597	0.364	0.221	97.245
1	8	1.53	0.1569	0.221	0.134	40.157
1	9	0.85	0.0321	0.134	0.081	16.450
1	10	0.476	0.0066	0.081	0.049	6.792
1	11	0.2721	0.0014	0.049	0.030	2.861
1	12	0.1474	0.0003	0.030	0.018	1.144
2	0	14.40	79.63	4.500	4.200	79.630
2	1	10.00	40.18	3.500	3.200	80.360
2	2	4.47	20.91	3.200	2.369	83.676
2	3	16.41	215.76	2.369	1.663	1726.126
2	4	11.38	73.71	1.663	1.167	1179.475
2	5	7.89	25.17	1.167	0.819	805.703
2	6	5.47	8.604	0.819	0.575	550.683
2	7	3.79	2.94	0.575	0.403	376.394
2	8	2.63	1.0048	0.403	0.283	257.221
2	9	1.824	0.3429	0.283	0.199	175.577
2	10	1.264	0.117	0.199	0.139	119.845
2	11	0.878	0.040	0.139	0.098	83.013
2	12	0.6082	0.0137	0.098	0.069	55.982
3	0	9.30	88.09	4.000	1.900	88.090
3	1	10.00	11.360	2.000	1.900	22.720
3	2	5.46	5.725	1.900	1.845	22.900
3	3	21.46	169.61	1.845	1.293	1356.948
3	4	15.86	61.53	1.293	0.906	984.538
3	5	11.72	22.33	0.906	0.635	714.684
3	6	8.66	8.102	0.635	0.445	518.587
3	7	6.40	2.939	0.445	0.312	376.226
3	8	4.73	1.067	0.312	0.219	273.195
3	9	3.50	0.3870	0.219	0.153	198.160
3	10	2.586	0.1404	0.153	0.107	143.778
3	11	1.912	0.0510	0.107	0.075	104.353
3	12	1.413	0.0185	0.075	0.053	75.707
On each kidney total volume of all branches			5983.64	5572.68		4879.88
Difference with plastic casts, B in Table 1			-24.64	-14.54		-20.95

Table 3. Measurements pertaining to the branches of the theoretical fractal trees of the three kidneys, when the fractal dimension is 1.94 and when, for input, the data from Branch 2 (the main branch of the renal artery situated between the second and third bifurcation) of their respective plastic casts is employed. Branch number 0 is renal artery, from its separation from the abdominal aorta artery to its first bifurcation into dorsal and ventral rami.

Kidney identification number			1	2		3
Maximun number of bifurcations from the renal artery to the glomerulus			12	12		12
Kidney number	Branch number	Length (mm)	Volume (mm^3)	Diameter of largest base (mm)	Diameter of smallest base (mm)	Total volume of all the branches equal to this (mm^3)
1	0	12.45	141.00	4.300	4.050	141.000
1	1	14.00	47.09	4.000	3.000	94.180
1	2	44.82	839.74	3.000	1.820	3358.996
1	3	25.08	173.13	1.820	1.105	1385.068
1	4	14.034	35.68	1.105	0.670	571.011
1	5	7.86	7.36	0.670	0.407	235.665
1	6	4.39	1.517	0.407	0.247	97.109
1	7	2.463	0.313	0.247	0.150	40.110
1	8	1.38	0.064	0.150	0.091	16.559
1	9	0.766	0.013	0.091	0.055	6.781
1	10	0.429	0.0027	0.055	0.033	2.799
1	11	0.245	0.0006	0.033	0.020	1.179
1	12	0.1329	0.0001	0.020	0.012	0.471
2	0	14.40	79.63	4.500	4.200	79.630
2	1	10.00	40.18	3.500	3.200	80.360
2	2	17.93	432.10	3.200	2.246	1728.406
2	3	12.439	147.67	2.246	1.576	1181.420
2	4	8.627	50.45	1.576	1.106	807.318
2	5	5.981	17.23	1.106	0.776	551.513
2	6	4.149	5.89	0.776	0.545	376.970
2	7	2.878	2.013	0.545	0.382	257.675
2	8	1.996	0.687	0.382	0.268	176.101
2	9	1.383	0.234	0.268	0.188	120.213
2	10	0.958	0.08	0.188	0.132	82.060
2	11	0.665	0.0274	0.132	0.093	56.158
2	12	0.461	0.0094	0.093	0.065	38.336
3	0	9.30	88.09	4.000	1.900	88.090
3	1	10.00	11.36	2.000	1.900	22.720
3	2	40.04	333.96	1.900	1.332	1335.844
3	3	29.60	121.20	1.332	0.933	969.666
3	4	21.88	43.98	0.933	0.654	703.742
3	5	16.18	15.96	0.654	0.458	510.989
3	6	11.95	5.79	0.458	0.321	370.876
3	7	8.837	2.102	0.321	0.225	269.128
3	8	6.536	0.736	0.225	0.158	195.472
3	9	4.829	0.277	0.158	0.111	141.814
3	10	3.568	0.1005	0.111	0.077	102.916
3	11	2.638	0.0365	0.077	0.054	74.711
3	12	1.949	0.0132	0.054	0.038	54.212
On each kidney total volume of all branches			5950.92	5536.15	4840.17	
Difference with plastic casts, B in Table 1				8.07	21.97	18.75

Table 4. Correlation coefficients when comparing data groups contained in Table 1 with that pertaining to kidney 1, 2 and 3 in Tables 2 and 3.

Table 1 data compared with	**length**			**Volume**			**Diameter of largest base**			**Diameter of smallest base**		
Kidney number	1	2	3	1	2	3	1	2	3	1	2	3
Table 2 data												
Lenght	0.71	0.89	0.44	-	-	-	-	-	-	-	-	-
Volume	-	-	-	0.15	0.24	0.17	-	-	-	-	-	-
Diameter of largest base	-	-	-	-	-	-	0.97	0.99	0.93	-	-	-
Diameter of smallest bsse	-	-	-	-	-	-	-	-	-	0.97	0.99	0.98
Table 3 data												
Lenght	0.06	0.82	0.38	-	-	-	-	-	-	-	-	-
Volume	-	-	-	0.29	0.33	0.35	-	-	-	-	-	-
Diameter of largest base	-	-	-	-	-	-	0.98	0.99	0.87	-	-	-
Diameter of smallest base	-	-	-	-	-	-	-	-	-	0.98	0.99	0.97

to the plastic casts (Table 1).

Although the programme carries out all its calculations using 8-digit real numbers, these are later rounded down to only 2, 3, or 4, as can be seen from the Tables.

Discussion

With the aid of the results that we have obtained, let us now return to the question we first posed at the beginning of this work: could the fractal dimension be that unique number which can serve to define an organ in a species? In order to answer this question, it is helpful to divide it into three. Is the arterial tree of the kidney a fractal organ? Does our method allow us to calculate the fractal dimension of an organ? Is the fractal dimension unique for an organ?

It is very difficult to answer the first of these questions because nobody has yet defined a fractal objectively. But we can say that the arterial tree of the kidney is a fractal organ, for three usual reasons: because of its aspect (Gleick, 1987; Goldberger and West, 1987; Goldberger et al., 1990); because we have been able (with the aid of our fractal programme) to construct theoretical fractal trees that have the same volume as the organ; and because an object is fractal if the pattern with which its structures have been constructed is also a fractal (Mandelbrot, 1987).

If the arterial tree of the kidney has a specific property, such as being fractal, this is important for the complete organ because «the structure of the vascular network in the organs is so characteristic that its aspect allows the diagnosis of the organ and even of the tissue to which it belongs» (Djavakhichvili and Komakhjidza, 1970).

With respect to the second question (does our method allow us to calculate the fractal dimension of an organ?) we can again answer this in the affirmative. The data contained in Tables 2-4 shows us that various fractal dimensions are possible for the template of each kidney, using our fractal programme.

As our fractal pattern we have chosen a *V*-shape for our fractal programme (Fig. 1) because it is very similar to the actual dichotomic bifurcations observed in the plastic casts.

We do not suggest that our fractal programme constructs an exact template of each plastic cast, because variations exist between each of these, as shown by the measurements taken from them (Table 1); we have been looking for similarities, rather than differences, with the aid of a model that simplifies reality, applying to our work what Lefevre (1983) has said with regards to the lung, namely «modeling the pulmonary arterial tree is considered as an optimal synthesis of the problem at the teleonomical optimization of a fractal model of the whole lung».

Our fractal programme carries out all its calculations with 8-digit real numbers, although later these are rounded down to maximum the first four decimal figures because this is the highest degree of precision that we can obtain when taking measurements directly from the plastic casts (Table 1). This is also the reason why the fractal dimensions used by the fractal programme change in increments of 0.01 units, which is the grade of sensibility; experimentally we observed less increments were not profitable.

The maximum of 12 levels of dichotomic bifurcations that we have counted in the three plastic casts of the kidneys represent 21^2 branches, corresponding in man to 2^{20} (Poirier, 1977). This data, drawn from only three kidneys, has no statistical value, but we have thought it was enough for this first work.

When comparing the volumes calculated by the fractal programme with respect to the volumes of the plastic casts, we allow for an error of ± 0.05%, a level which is habitually accepted in biological research. The volume of both the efferent arterioles (Fig. 8) and of the glomeruli are excluded in the volume of the plastic casts, whilst the theoretical volumes are calculated down to the glomerulus.

But can we say that the fractal dimension of an organ is unique? With the data that is now available to us (Table 1) and with the calculations of our fractal programme, we consider that the most representative number is only one which appears in all results of three kidneys; namely 1.94 (Table 2).

According to our calculation system, only one set of measurements is necessary, but for the purpose of this investigation we used the 13 sets one after the other, in order to determine which of them provided the most useful data.

If we accept this value of 1.94 as valid, this means that the true fractal dimension of the skeleton of the fractal tree (which appears as a line in Fig. 2) is 1.39 ($\approx \sqrt{1.94}$) because, in the calculation (see Appendix), the fractal dimension has only intervened in two of the three lengths (height and radius) on each truncated cone (Fig. 4) which form part of the theoretical fractal tree. Therefore the fractal dimension of the fractal tree as a whole is 2.7 ($\approx 1.39^3$), a value which is similar to those resulting from the application of other methods. Thus arterial and venous casts of a kidney have a fractal dimension of between 2 and 3 (Sernetz et al., 1985), as does the lung alveolar-capillary (Goldberger and West, 1987). The yolk vessels of a chicken embryo incubated for 4 days have a fractal dimension of 5/3 (Tsonis and Tsonis, 1987).

Using the value of 1.94, and once again employing our fractal programme, we have calculated the measurements of the Branches of the fractal trees corresponding to the three kidneys, using as input the data from the plastic casts of either Branch 1 or Branch 2 respectively (Tables 2, 3). These measurements have a very high correlation (Table 4) with their equivalents from the plastic casts (Table 1), a fact which we have interpreted as support for the method that we have used to calculate and select the figure of 2.7 as the fractal dimension of the kidney.

We suggest that the technique that we have constructed and, we believe, proved in this work, will be useful in carrying out more extensive and systematic statistical studies on fractal morphometrics, using either our fractal programme or a similar one. These extended studies will allow for the calculation of the fractal dimension not only of the kidney but also of other organs, thus enabling us to prove whether these are equal or different; later these studies could be expanded to include those pathologies originating from arterial alterations. If the results obtained reveal significantly different values, then we will have available to us a new and more objective taxonomy.

Appendix

An object is fractal if the pattern with which its structure has been constructed is also a fractal (Mandelbrot, 1987). The pattern we have chosen is a *V*-shaped fractal (Fig. 1) because it is similar to the actual dichotomic bifurcations which we observed on the plastic casts. The repetition of the pattern at different scales forms the skeleton of the fractal tree (Figs. 2, 3).

Calculation of the fractal dimension of our pattern

** In the calculation of the fractal dimension we have employed the Hausdorff-Besicovitch formula (Abbot, 1981):

$$\mathbf{L} = \mathrm{L}\,(\Delta \mathrm{X})^{\mathrm{D}-1} \text{ where}$$

L is the Hausdorff invariant length, so called because it always has the same value for each pattern, independent of the scale of the fractal pattern *V* (Fig. 1).

L is a part of the total length of the fractal tree, made up by adding together the lengths of all the fractal patterns whose arms have a value equal to ΔX.

ΔX is the length of each arm, or branch, of the fractal pattern (*V*) according to the resolution used in each case.

D is the fractal dimension

•• The fractal pattern is constructed by way of a basic line (Fig. 9) where: $L_o = \Delta X_o$

• In Figure 3 the Hausdorff-Besicovitch formula (Abott, 1981) gives:

$$\mathrm{L} = \mathrm{L_o} * (\Delta \mathrm{X_o})^{\mathrm{D}-1} \quad (1)$$

(In this and in all subsequent formulae * represents the multiplication sign).

•• In Figure 10 we can see how the initial straight line (Fig. 9) appears after the application, for the first time, of the fractal pattern (*V*) (Fig 1), where:

$$\mathrm{L_1} = 2 * \Delta \mathrm{X_1}$$

$$\Delta \mathrm{X}_1^2 = \left[\frac{\mathrm{L_o}^2}{\mathrm{m}^2}\right] + \left[\frac{\mathrm{L_o}^2}{\mathrm{m}^2}\right] = 2 * \left[\frac{\mathrm{L_o}^2}{\mathrm{m}^2}\right]$$

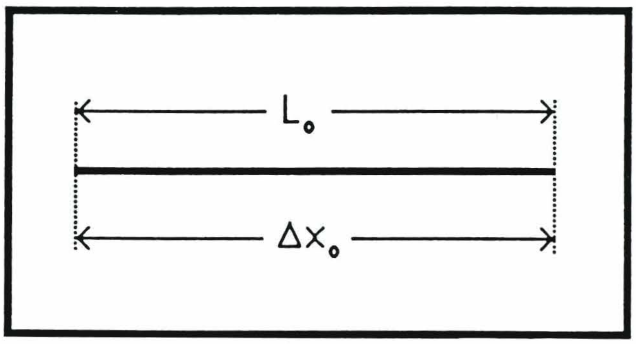


Fig. 9. Starting line for the calculation of the fractal dimension (Hausdorff-Besicovith dimension). L_o is the initial length of the fractal tree when the only arm, or branch, of the fractal pattern measures X_o. ΔX_o is the initial length of the fractal pattern.

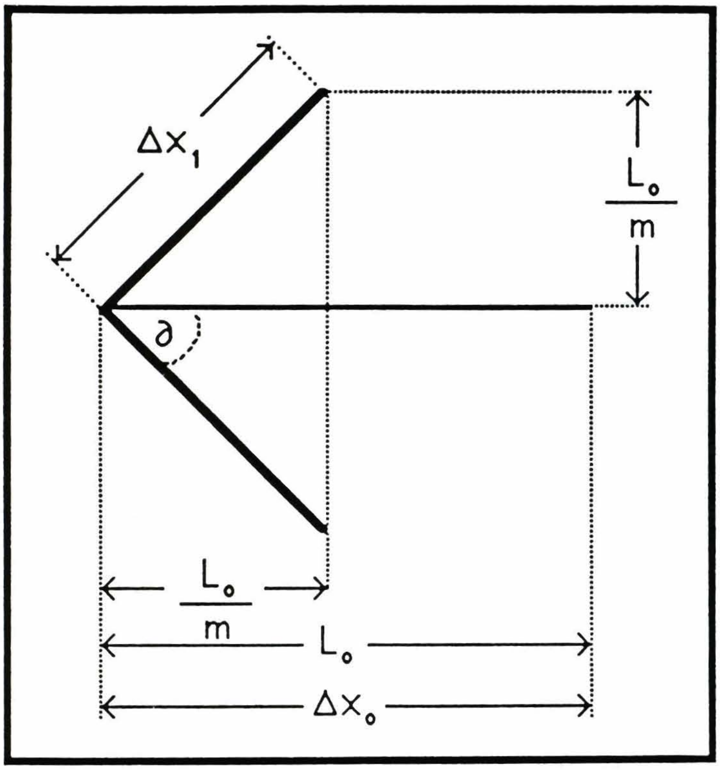


Fig. 10. *V*-shaped fractal model and its measurements after the first bifurcation. L_o is the initial length of the fractal tree when the only arm, or branch, of the fractal pattern measures ΔX_o . ΔX_o is the initial length of the fractal pattern. ΔX_1 is the initial length of each arm, or branch, of the fractal pattern after the first bifurcation. m is a fixed value which makes L_o become smaller in successive ramifications. ∂ is half the value of the angle formed by the two arms of the fractal pattern.

where: m is a fixed value for each fractal tree constructed by the programme; m makes L_O become smaller and, in this way, the scaling of the fractal pattern changes in succesive ramifications.

$$\Delta X_1 = \sqrt{2} * \left[\frac{L_O}{m}\right] = \sqrt{2} * \left[\frac{\Delta X_O}{m}\right]$$

$$L_1 = 2 * \sqrt{2} * \left[\frac{L_O}{m}\right]$$

• We apply the Hausdorff-Besicovith formula (Abbott, 1981)

$$L = L_1(\Delta X_1)^{D-1} = \left(2 * \sqrt{2} * \left[\frac{L_o}{m}\right]\right) * \left(\sqrt{2} * \left[\frac{\Delta X_o}{m}\right]\right)^{D-1} \qquad (2)$$

• • To calculate D, which is the fractal dimension of our pattern we divide formula (2) by (1);

$$\frac{L}{L} = \frac{\left(2 * \sqrt{2} * \left[\frac{L_o}{m}\right]\right) * \left(\sqrt{2} * \left[\frac{\Delta X_o}{m}\right]\right)^{D-1}}{L_o * (\Delta X_o)^{D-1}}$$

simplifying

$$1 = \left(2 * \sqrt{2} * \left[\frac{1}{m}\right]\right) * \left(\sqrt{2} * \left[\frac{1}{m}\right]\right)^{D-1}$$

we now take logarithms (log)

$$0 = \log\left(2 * \sqrt{2} * \left[\frac{1}{m}\right]\right) + \left[(D-1) * \log\left(\sqrt{2} * \left[\frac{1}{m}\right]\right)\right]$$

simplifying

$$D = -\frac{\log 2}{\log\left(\frac{\sqrt{2}}{m}\right)}$$

• As D to have significance, must be greater than zero, the programme only needs to calculate those values where m is greater than Thus: $\sqrt{2}(\sqrt{2} = 1.414213562)$.

$$D = -\frac{1.414213562}{\log\left(\frac{1.414213562}{m}\right)}$$

• • Each fractal tree constructed by the programme is different from the rest in the fractal dimension of its pattern and this is because m changes. The changes to m give rise to changes in the length of the arms for each *V*-shaped fractal form (Figs. 1, 2).

The construction of volumes associated with the fractal pattern

The programme constructs fractal trees by the repetition of the *V*-shaped fractal pattern (Fig. 1) at different scales (Figs. 2, 3). Whilst the plastic cast is tridimensional and has a volume, the *V*-shaped form does not. In order to obtain volumes for the programme to compare, it is necessary to create them from each fractal tree; to that end, the programme uses each arm, or branch, of the *V*-shape as a skeleton or control axis around which to construct a truncated cone (Fig. 4).

The programme calculates the total volume of each fractal tree which it has constructed by totalling the volumes of all the truncated cones, which have themselves been constructed around the fractal tree, this tree having been formed by the union of *V*-shaped fractal patterns (Fig. 1) repeated at different scales (Figs. 2, 3).

The volume of each truncated cone is:

According to the formula Vis the volume of the

$$V = \frac{1}{3} * \left[\pi * h * \left(R^2 + r^2 + R * r\right)\right]$$

truncated cone, h is its height, R the radius of the largest base, and r that of the smallest base; * is the multiplication sign, and $\pi = 3.141592654$.

Two truncated cones, constructed around the two arms, or branches, of a V-shaped fractal meet at a point (Fig. 4A, B); a third truncated cone, arising from an earlier ramification also meets at that point. The connection between these three truncated cones gives rise to a geometrical body (Fig. 4A, B) whose volume we have calculated by way of aleatory Montecarlo type system.

$V = 0.896\ r^3$

In this formula V represents the volume of the geometrical body made up by the connection of the three truncated cones. In the same geometrical body r represents the height and the radius of the largest base (Fig. 4B).

In the calculations performed by the programme R and r are always related in the successive branches of the fractal trees by a coefficient equivalent to:

(R - r) * *Proportional Factor to Dimension Fractal* where *Proportional Factor to Dimension Fractal* ≈ the relationship of length which exists between the arms of any two successive ramifications, ΔX_n of the fractal tree which has been constructed. Its value is:

$$\textit{Proportional Factor to Dimension Fractal} = \left[\frac{\sqrt{2}}{m}\right]^r$$

n is the ordinal number of the bifurcation considered (1, ..., 12).

m is a fixed value for each fractal tree constructed by the programme, which causes the length of the arms of the fractal pattern to became smaller, and thus its scaling changes in each successive ramification.

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