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# **Effective Field Theory of Interactions on the Lattice**

Received: 31 March 2015 / Accepted: 25 April 2015 © Springer-Verlag Wien 2015

**Abstract** We consider renormalization of effective field theory interactions by discretizing the continuum on a tight-binding lattice. After studying the one-dimensional problem, we address s-wave collisions in three dimensions and relate the bare lattice coupling constants to the continuum coupling constants. Our method constitutes a very simple avenue for the systematic renormalization in effective field theory, and is especially useful as the number of interaction parameters increases.

# **1** Introduction

The physics of quantum systems with strong interparticle interactions is under intense study on several fronts as we are currently experiencing a convergence of different areas and methods that hold promise to yield important insights into this crucial problem. Experiments at RHIC and LHC are expected to provide new details on the strong interaction dynamics of nuclear and particle physics [1], while advances in atomic, molecular and optical physics have opened up the regime of strong interactions for many-body physics with atoms and molecules [2,3]. The latter may be used to mimick the conditions found in strongly-interacting condensed-matter environments [4,5], or even neutron stars [6–8]. There is great hope that these developments will complement and cross-fertilize each other in coming years [9].

The theoretical study of systems with strong interactions can be extremely complicated as any naïve perturbative approach is doomed to fail. However, the tremendous increase in computational power and the development of new methods means that one can now perform simulations on discretized lattices of strongly-interacting few- and many-body dynamics that are not only extremely precise but also yield controlled estimations of uncertainties [10,11]. The results of such simulations can subsequently be extrapolated to the limit of zero lattice spacing and predictions for continuum dynamics may be obtained. To utilize these powerful simulation tools it is crucial to have a fully consistent description of two-body interaction terms on a lattice, and furthermore be able to connect this knowledge to parameters that are often defined and measured in free space. Examples are the scattering length or effective range parameters that describe low-energy scattering and have a long history in nuclear and atomic physics.

Within the realm of effective field theory (EFT), the simulation techniques discussed above are extremely useful for studying low-energy dynamics in the presence of strong interactions. EFT originates from Weinberg's expansion of the nucleon–nucleon potential at low momenta (compared to the nucleon mass) with chiral

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perturbation theory [12,13]. There are issues with regularization and renormalization of this expansion that have to be treated with great care, particularly when higher-order terms are included [14–16]. Furthermore, these issues become more intricate on a lattice as discussed in the seminal work of Lüscher [17–19].

In this paper we consider regularization and renormalization of two-body interactions in EFT and provide some solvable, physically important, and technically simpler models than those that are typically discussed. We start in one-dimensional settings where the procedures are particularly clear, and so far unexplored. This is done for both *s*- and *p*-wave effective interactions on a lattice for which the continuum limit may be performed in the end. One-dimensional systems are realizable with cold atoms [2] and strongly interacting one-dimensional gases have been produced in recent years to study both many-body [20-23] and few-body physics [24,25]. After discussing the one-dimensional renormalization on the lattice, we proceed to the three-dimensional case and demonstrate how the lessons learned in one dimension may be used, at zero extra cost, to renormalize the interactions in a much more straightforward manner than the direct approach via the Lippmann–Schwinger equation. We also provide the direct link between the discretized and continuum EFT parameters, especially important for the three-dimensional case.

#### **2** Effective Field Theory

In EFT of scattering, the momentum representation of the two-body interaction,  $V(\mathbf{k}, \mathbf{k}')$ , is expanded in series as

$$V(\mathbf{k}, \mathbf{k}') = g_0 + \frac{g_2}{2} \left( \mathbf{k}^2 + \mathbf{k}'^2 \right) + g_{11} \mathbf{k} \cdot \mathbf{k}' + \dots$$
(1)

In three spatial dimensions (3D), the above interaction is renormalizable, at least to order  $k^2$ , and yields the correct s-wave scattering amplitude or, equivalently, the phase-shifts, at low energy. That is, the effective range expansion

$$k \cot(\delta_0) = -\frac{1}{a} + \frac{r}{2}k^2 + \dots,$$
(2)

where a and r are, respectively, the s-wave scattering length and effective range, while  $\delta_0$  is the s-wave phase-shift.

The renormalization of the interaction in Eq. (1) has been carried out by solving the Lippmann–Schwinger equation for the T-matrix [26–28]. This procedure, while conceptually simple, disembarks in frustratingly tedious algebra which masks the physics of the problem considerably. We here address the problem by discretizing the Schrödinger equation on a lattice. This method largely simplifies the problem and allows a clear interpretation of the results. Before considering the three-dimensional problem, we study one-dimensional scattering for which the renormalization prescription has not been elucidated yet. We also solve the problem of 1D "p-wave" interactions, where universal single-parameter renormalization does work [29, 30], in contrast with the 3D case where it does not [31, 32].

### **3** One-Dimensional Collisions

We define a 1D tight-binding lattice with two particles that can occupy discrete positions x = jd, where j is an integer and d is the lattice spacing. The lattice kinetic energy dispersion is chosen to be

$$E(k) = -2J\cos(kd) + 2J,$$
(3)

which is quadratic in k close to the continuum limit  $(d \rightarrow 0)$ . In momentum space, an interaction with a low-energy Taylor series given by Eq. (1) while respecting the periodicity of the Brillouin zone can be chosen as

$$V(k, k')/d = U + 2V \cos[(k - k')d].$$
(4)

## 3.1 s-wave Interaction

We first handle the s-wave part of the EFT interaction, that is, the renormalization of the coupling constants  $g_0$  and  $g_2$ . The lattice Hamiltonian corresponding to the energy dispersion (3) and the interaction (4) is given by

$$-J\sum_{j} \left( b_{j+1}^{\dagger} b_{j} + b_{j}^{\dagger} b_{j+1} \right) + \sum_{j} \left[ \frac{U}{2} n_{j} (n_{j} - 1) + V n_{j} n_{j+1} + 2J n_{j} \right].$$
(5)

Above,  $b_j (b_j^{\dagger})$  is the bosonic annihilation (creation) operator at the lattice site j, and  $n_j = b_j^{\dagger} b_j$  is the number operator. We have chosen to work with bosons for simplicity, but the following applies to spin-1/2 fermions, too. It must be noted that, with fermions, the interaction in Eq. (4) must be made spin-dependent, for otherwise the s-wave renormalized interaction would imply a hard-core "p-wave" interaction for triplet fermions, as we will see.

The two-body phase-shift for the Hamiltonian (5) is given by [33]

$$-\sin(kd)\cot\delta_0 = \frac{2u + [4\cos(kd) + u]v\cos(kd)}{uv - 4[2 - v\cos(kd)]},$$
(6)

where we have defined the dimensionless constants u = U/J and v = V/J. To take the continuum limit, we expand the trigonometric functions of kd to second order in kd. The phase-shift therefore becomes

$$-k \cot \delta_0 = \frac{1}{d} \frac{2u + 4v + uv - vk^2 d^2 (4+u)/2}{uv + 4v - 8}.$$
(7)

Above, we have assumed, in order not to have any  $k^2$ -dependence in the denominator, that  $vd^2 \rightarrow 0$  as  $d \rightarrow 0$ . We will see, after renormalization, that the assumption is indeed correct. Using the 1D effective range expansion,  $-k \cot \delta_0 = \alpha + \beta k^2$ , where  $\alpha = 1/a$  and  $\beta = -r$ , we find

$$u = -4\frac{\alpha d^2 + 2\beta}{d + 2\beta} \tag{8}$$

$$v = 4 \frac{\beta}{d - d^2 \alpha}.$$
(9)

Close to the continuum limit we have  $v \sim 1/d$ , and therefore our assumption that  $vd^2 \rightarrow 0$  is correct.

We now proceed to relate the lattice parameters u and v to the continuum EFT bare coupling constants  $g_0$  and  $g_2$  of Eq. (1). Close to the continuum limit, the lattice potential (4) is given by

$$V(k,k')/d = U + 2V - V[(k-k')d]^2 + O[(k-k')^4d^4].$$
(10)

We then equate Eq. (1) to (10) in the s-wave sector, obtaining

$$U + 2V = g_0/d \tag{11}$$

$$-Vd^2 = \frac{g_2}{2d}.$$
 (12)

The tunnel coupling constant J close to the continuum limit has to be adjusted in such a way that the energy dispersion, Eq. (3) is given by  $\hbar^2 k^2/2m$ . Therefore, J has the form

$$J = \frac{\hbar^2}{2md^2} \tag{13}$$

as  $d \rightarrow 0$ . The explicit expressions for the continuum coupling constants are given by

$$g_0 = \frac{\hbar^2}{2md} \left[ \frac{8\beta}{d - d^2\alpha} - 4\frac{d^2\alpha + 2\beta}{d + 2\beta} \right]$$
(14)

$$g_2 = -4\frac{\hbar^2\beta}{m(1-d\alpha)}.$$
(15)

The behavior of the coupling constants to leading order is easily recognized as  $g_0 \sim d^{-2} + O(d^{-1})$  and  $g_2 \sim 1$ .

#### 3.2 p-wave Interactions

We now renormalize the p-wave coupling constant  $g_{11}$ . This problem, which is simpler than the for s-wave interactions, has already been solved (see [30]) by comparing the action of the finite-difference kinetic energy operator on a fermionic wave function with Cheon–Shigehara boundary conditions [29] in the continuum.

We recover the renormalization result for  $g_{11}$ , and give a simple physical interpretation here by using the phase-shift in Eq. (6), which becomes the phase-shift for two polarized fermions in the limit  $U \rightarrow \infty$  [33], in virtue of the Bose–Fermi mapping theorem [34]. Close to the continuum limit, the phase-shift becomes

$$-k\cot\delta_1 = \frac{1}{d} \frac{2 + \nu(1 - k^2 d^2/2)}{\nu}.$$
(16)

Matching the relation above to the effective range expansion

$$-k\cot\delta_1 = \alpha + \beta k^2,\tag{17}$$

we obtain  $\beta = -d/2$ , and

$$v = -\frac{2}{1 - d\alpha}.\tag{18}$$

Interestingly, setting  $\beta = -d/2$  implies  $u \to \infty$ , too. Therefore, we can allow interactions to be spinindependent for hard-core spin-1/2 fermions with nearest-neighbor interaction. This, on the other hand, would imply a magical matching of s-wave and p-wave scattering amplitudes and therefore, even in the hard-core case, spin-dependent interactions would be strongly preferred based on physical grounds. In the case of finite u, however, the nearest-neighbor interaction, Eq. (8), diverges as  $v \sim d^{-1}$  and spin-independent interactions would lead to a nearest-neighbor hard-core condition for pairs of triplet fermions.

Finally, the bare coupling constant  $g_{11}$  is renormalized as

$$g_{11} = \frac{\hbar^2 v d}{m} = -\frac{2\hbar^2}{m} \frac{d}{1 - d\alpha},$$
(19)

which vanishes as  $d \to 0$ , but its dependence on the lattice spacing through  $d\alpha$  must be kept in all calculations before taking the continuum limit. This is the exactly same situation we observe in the single-parameter EFT in three-dimensions, and we will see in the next section that these two problems are formally equivalent. Clearly, a linearly vanishing coupling constant  $g_{11}$  tells us that the bare continuum limit has a linear UV divergence.

We now consider the next-to-leading order correction for p-wave scattering in 1D. This problem, which is relevant per se, can actually be used to renormalize the s-wave bare EFT parameters  $g_0$  and  $g_2$  in three dimensions, as we will show in the next section.

To solve the problem, we first invoke the Bose–Fermi mapping theorem so that we do not need to work with fermions. The Hamiltonian H' of the system is thus given by Eq. (5) with  $U \to \infty$  and an extra term, as

$$H' = H + W \sum_{j} n_{j} n_{j+2}.$$
 (20)

The phase-shift is easily obtained by using the methods of [35] for arbitrary finite-range interactions on a 1D lattice. Defining v = V/J and w = W/J, the phase-shift reads

$$-\cot \delta_1 = \frac{4 + 2(v+w)\cos(kd) + vw\cos(2kd) + 2w\cos(3kd)}{2(v+2w+vw\cos(kd) + 2w\cos(2kd))\sin(kd)}.$$
(21)

Assuming now that  $(vw + 2w)d^2 \rightarrow 0$  as we approach the continuum limit, and fitting v and w to match the effective range expansion, we obtain

$$\alpha = \frac{1}{d} \frac{4 + 2v + 4w + vw}{2v + 8w + 2vw}$$
(22)

$$\beta = -d(v + 2vw + 10w), \tag{23}$$

The lattice interaction parameters can have two different values each, denoted as  $v_{\pm}$  and  $w_{\pm}$ , given by

$$v_{\pm} = \frac{1}{2d(2d\alpha - 3)} \left( 24d - 12d^2\alpha - \beta + 2d\alpha\beta \pm \Gamma \right)$$
(24)

$$w_{\pm} = \frac{1}{4d(2d\alpha - 1)} \left( 8d - 12d^2\alpha + \beta - 2d\alpha\beta \pm \Gamma \right)$$
(25)

where we have defined

$$\Gamma = \sqrt{16d(1 - 2d\alpha) [d(2 + \alpha\beta) - \beta] + [\beta - 2d(6d\alpha + \alpha\beta - 4)]^2}.$$
(26)

We can now see that  $(v_{\pm}w_{\pm} + 2w_{\pm}) = (\beta \pm |\beta|)d + O(d^2)$ , and therefore our assumption was correct. The above expressions for the lattice coupling constants is now more complicated than in the previous cases. We will see how this is completely equivalent to three-dimensional s-wave EFT when the parameters are properly re-defined in the following section.

#### **4** Three-Dimensional Collisions

In three spatial dimensions, there is a rather intrincate relation [26,28] between the bare EFT parameters  $g_0$  and  $g_2$  and the scattering length a and effective range r to renormalize the two-body problem in the s-wave sector. Typically, this relation is obtained by solving the corresponding Lippmann–Schwinger equation non-perturbatively and, as we mentioned earlier, involves rather cumbersome and tedious algebraic manipulations. We here use the results of the previous section to obtain these relations in a straightforward way.

We begin with the reduced radial Hamiltonian in the s-wave channel

$$H = -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + V(r).$$
 (27)

The eigenfunctions of *H* are the reduced radial wave functions u(r) = rR(r), where R(r) are the radial wave functions of the original radial Hamiltonian. The discretization of *H* is trivial, and we only need to implement the boundary condition u(r < 0) = 0 for the reduced wave functions. A little care must be taken in using Dirac delta functions now. To illustrate this, we look at the bare delta interaction  $V_D$ , which has the form

$$V_D(r) = g_0 \frac{\delta(r)}{4\pi r^2}.$$
(28)

Direct discretization of the above interaction, if we allow the point j = 0 to be included on the lattice, is ambiguous. However,  $r = jd \rightarrow 0$  for any finite j. Therefore, we can simply define the lattice coordinate space for  $j \ge 1$  without loss of generality, and implement the boundary condition u(0) = 0 before the continuum limit is taken. The discretized version of the Hamiltonian (27) with nearest and next-nearest neighbor interactions on the lattice is therefore exactly given by the 1D hard-core (or fermionic) Hamiltonian of Eq. (20). The phase-shifts are given by Eq. (21). The only difference now lies in adjusting the EFT bare parameters  $g_0$  and  $g_2$  for the three-dimensional case.

The Fourier transform of the s-wave part of the interaction in continuous space is given by

$$V_{S}(q) = 4\pi \int_{0}^{\infty} dr r^{2} j_{0}(qr) V(r), \qquad (29)$$

where  $q = |\mathbf{k} - \mathbf{k}'|$ , and  $j_0(x) = \sin(x)/x$  is the zero-th order spherical Bessel function. We take the lowenergy expansion of  $V_S$  on the left hand side,  $V_S(q) = g_0 + g_2 q^2/2 + O(q^4)$ . Then, we discretize the Fourier transform for our lattice problem (note that this is not the same as using the discrete Fourier transform) using Riemann's sums, as

$$V_{\text{lattice}}(q) = 4\pi d^3 \left[ V j_0(qd) + 4W j_0(2qd) \right].$$
(30)

Expanding the above expression to fourth order in (kd), and equating the resulting expansion to the continuum limit, we find

$$g_0 = 4\pi \left( V + 4W \right) d^3, \tag{31}$$

$$\frac{g_2}{2} = -4\pi \left(\frac{V}{6} + \frac{W}{3}\right) d^5,$$
(32)

$$d^7 \left( \frac{V}{120} + \frac{8W}{15} \right) \to 0. \tag{33}$$

The effective range expansion in three-dimensions looks exactly like its one-dimensional counterpart, and therefore the lattice coupling constants are here given by Eqs. (24) and (25). The EFT coupling constants  $g_0$  and  $g_2$  follow immediately from Eqs. (24, 25, 31, 32).

The only point we have to address now, in order to be able to use the EFT interaction in continuous space, is to relate the lattice spacing d to the cut-off  $\Lambda$ . Note that this is unnecessary in 1D, where we can do all relevant calculations on the lattice at the N-body level. We can set W = 0 and obtain the bare coupling constant  $g_0$  as a function of the lattice spacing and the scattering length  $a = 1/\alpha$ . We obtain

$$g_0 = \frac{2\pi\hbar^2 d}{m} \frac{2}{d/a - 1}.$$
(34)

From standard cut-off renormalization, we have

$$g_0 = \frac{4\pi\hbar^2 a}{m} \frac{1}{1 - 2a\Lambda/\pi}.$$
(35)

Comparing now Eqs. (34) and (35), we see that  $d = \pi/2\Lambda$ , which is the desired relation.

## **5** Conclusions and Outlook

We have presented a simple way of renormalizing two-body EFT interactions by using a discrete lattice and taking the correct continuum limit. In one dimension, the lattice discretization can be used for the many-body problem, where numerically accurate methods, such as the density matrix renormalization group (for a review, see [36]) are available. Therefore, our renormalization prescriptions can be used to obtain effective-range corrections in an essentially exact fashion. In the three-dimensional case, we discretized the s-wave radial Schrödinger equation, and used the equivalent one-dimensional results to obtain the EFT coupling constants in a straightforward manner.

Our results can be extended to include a finite, arbitrary number of coupling constants [35], and may be useful to clarify, whithout the tedious intricacies of the multichannel Lippmann–Schwinger equation in continuous space, whether quartic s-wave interactions in three dimensions are renormalizable [28,37]. Extension of our method to higher angular momentum channels may be possible by using a mixed lattice-continuous representation of the partial-wave radial Schrödinger equation. Of special relevance is the problem of twoparameter renormalization of p-wave interactions, which is important for highly polarized Fermi gases [32,38] and p-wave superfluids [31].

### References

- 1. Braun, J.: Fermion interactions and universal behavior in strongly interacting theories. J. Phys. G 39, 033001 (2012)
- 2. Bloch, I., Dalibard, J., Zwerger, W.: Many-body physics with ultracold gases. Rev. Mod. Phys. **80**, 885 (2008)
- Chin, C., Grimm, R., Julienne, P., Tiesinga, E.: Feshbach resonances in ultracold gases. Rev. Mod. Phys. 82, 1225 (2010)
   Lewenstein, M. et al.: Ultracold atomic gases in optical lattices: mimicking condensed matter physics and beyond. Adv.
- Phys. 56, 243 (2007)
  5. Esslinger, T.: Fermi-hubbard physics with atoms in an optical lattice. Annu. Rev. Condens. Matter. Phys. 1, 129 (2010)
- 6. Baker, G.A. Jr.: Neutron matter model. Phys. Rev. C 60, 054311 (1999)
- 7. Schwenk, A., Pethick, C.J.: Resonant Fermi Gases with a Large Effective Range. Phys. Rev. Lett. 95, 160401 (2005)
- 8. Carlson, J., Morales, J. Jr., Pandharipande, V.R., Ravenhall, D.G.: Quantum Monte Carlo calculations of neutron matter. Phys.
- Rev. C. 68, 025802 (2003)
  9. Zinner, N.T., Jensen, A.S.: Comparing and contrasting nuclei and cold atomic gases. J. Phys. G: Nucl. Part. Phys. 40, 053101 (2013)

- 10. Lee, D.: Lattice simulations for few- and many-body systems. Prog. Part. Nucl. Phys. 63, 117 (2009)
- Endres, M.G., Kaplan, D.B., Lee, J.-W., Nicholson, A.M.: Lattice Monte Carlo calculations for unitary fermions in a finite box. Phys. Rev. A 87, 023615 (2013)
- 12. Weinberg, S.: Nuclear forces from chiral lagrangians. Phys. Lett. B. 251, 288 (1990)
- 13. Weinberg, S.: Effective chiral lagrangians for nucleon-pion interactions and nuclear forces. Nucl. Phys. B 363, 3 (1991)
- 14. Phillips, D.R., Beane, S.R., Cohen, T.D.: Regularization and renormalization in effective field theories of the nucleon-nucleon interaction. Nucl. Phys. A. 631, 447 (1998)
- 15. Kaplan, D.B., Savage, M.J., Wise, M.B.: A new expansion for nucleon-nucleon interactions. Phys. Lett. B. 424, 390 (1998)
- 16. Kaplan, D.B., Savage, M.J., Wise, M.B.: Two-nucleon systems from effective field theory. Nucl. Phys. B. 534, 329 (1998)
- 17. Lüscher, M.: Volume dependence of the energy spectrum in massive quantum field theories: I. Stable particle states. Commun. Math. Phys. **104**, 177 (1986)
- Lüscher, M.: Volume dependence of the energy spectrum in massive quantum field theories: II. Scattering states. Commun. Math. Phys. 105, 153 (1986)
- 19. Lüscher, M.: Two-particle states on a torus and their relation to the scattering matrix. Nucl. Phys. B 354, 531 (1991)
- 20. Paredes, B. et al.: Tonks-Girardeau gas of ultracold atoms in an optical lattice. Nature (London) 429, 277 (2004)
- 21. Kinoshita, T., Wenger, T., Weiss, D.: Observation of a one-dimensional tonks-girardeau gas. Science 305, 1125 (2004)
- 22. Kinoshita, T., Wenger, T., Weiss, D.: Local pair correlations in one-dimensional bose gases. Phys. Rev. Lett. 95, 190406 (2005)
- 23. Haller, E. et al.: Realization of an excited, strongly correlated quantum gas phase. Science 325, 1224 (2009)
- 24. Serwane, F. et al.: Deterministic preparation of a tunable few-fermion system. Science **332**, 336 (2011)
- 25. Zürn, G. et al.: Fermionization of two distinguishable fermions. Phys. Rev. Lett. 108, 075303 (2012)
- Phillips, D.R., Beane, S.R., Cohen, T.D.: Nonperturbative regularization and renormalization: simple examples from nonrelativistic quantum mechanics. Ann. Phys. (NY). 263, 255 (1998)
- 27. Frederico, T., Delfino, A., Tomio, L.: Renormalization group invariance of quantum mechanics. Phys. Lett. B 481, 143 (2000)
- 28. Braaten, E., Kusunoki, M., Zhang, D.: Scattering models for ultracold atoms. Ann. Phys. (NY) **323**, 1770 (2008)
- Cheon, T., Shigehara, T.: Fermion-boson duality of one-dimensional quantum particles with generalized contact interactions. Phys. Rev. Lett. 82, 2536 (1999)
- 30. Muth, D. Fleischhauer M., Schmidt, B.: Discretized versus continuous models of *p*-wave interacting fermions in one dimension. Phys. Rev. A 82, 013602 (2010)
- 31. Gurarie, V., Radzihovsky, L.: Resonantly paired fermionic superfluids. Ann. Phys. (NY) 322, 2 (2007)
- 32. Levinsen, J., Massignan, P., Chevy, F., Lobo, C.: p-Wave polaron. Phys. Rev. Lett. 109, 075302 (2012)
- Valiente, M., Petrosyan, D.: Scattering resonances and two-particle bound states of the extended Hubbard model. J. Phys. B: At. Mol. Opt. Phys. 42, 121001 (2009)
- Girardeau, M.D.: Relationship between systems of impenetrable bosons and fermions in one dimension. J. Math. Phys. 1(6), 516–523 (1960)
- 35. Valiente, M.: Lattice two-body problem with arbitrary finite-range interactions. Phys. Rev. A 81, 042102 (2010)
- 36. Hallberg, K.A.: New trends in density matrix renormalization. Adv. Phys. 55, 477 (2010)
- 37. Yang, J.-F., Huang, J.-H.: Renormalization of NN scattering: contact potential. Phys. Rev. C 71, 034001 (2005)
- 38. Chevy, F., Mora, C.: Ultra-cold polarized Fermi gases. Rep. Prog. Phys. 73, 112401 (2010)