



Inconsistency of the data on the $K_1(1270) \rightarrow \pi K_0^*(1430)$ decay width

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ABSTRACT

We show, using the same Lagrangian for the $K_1(1270) \rightarrow \pi K_0^*(1430)$ and $K_0^*(1430) \rightarrow K_1(1270)\pi$ decays, that the present PDG data on the partial decay width of $K_1(1270) \rightarrow \pi K_0^*(1430)$ implies a width for $K_0^*(1430) \rightarrow K_1(1270)\pi$ decay which is about one order of magnitude larger than the total $K_0^*(1430)$ width. A discussion on this inconsistency is done, stressing its relationship to the existence of two $K_1(1270)$ states obtained with the chiral unitary theory, which are not considered in the experimental analyses of $K\pi\pi$ data.

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Data on $K\pi\pi$ produced in high energy diffractive Kp and Kd collisions have been analyzed in the past and the $K_1(1270)$ and $K_1(1400)$ states were identified more than forty years ago, together with their decay channels [1,2]. The $K^*\pi$ and ρK decay modes are the most prominent ones but a surprisingly large experimental value for the branching fraction for the $K_1(1270) \rightarrow \pi K_0^*(1430)$ ($\pi\kappa$ in the past) appears. A reanalysis of these data is done in Ref. [3] and the PDG [4] quotes it as giving

$$\Gamma_1 \equiv \Gamma[K_1(1270) \rightarrow \pi K_0^*(1430)] = 26 \pm 6 \text{ MeV}, \quad (1)$$

for a K_1 with mass and width given by

$$M_{K_1} = 1253 \pm 7 \text{ MeV}, \quad \Gamma(K_1(1270)) = 90 \pm 20 \text{ MeV}. \quad (2)$$

However, it is stated in the PDG that this partial decay width is “not used for averages, fits, limits, etc.” On the other hand the only data not excluded for “averages, fits, limits, etc.” are those from Ref. [5], with

$$\text{BR}[K_1(1270) \rightarrow \pi K_0^*(1430)] = (28 \pm 4)\%, \quad (3)$$

which is a big number as we shall see.

Furthermore, there is a much more recent experiment from Belle [6], which finds a significantly smaller branching ratio

$$\text{BR}[K_1(1270) \rightarrow \pi K_0^*(1430)] = (2.0 \pm 0.6)\%, \quad (4)$$

but, however, once again this datum is “not used for averages, fits, limits, etc.” by the PDG. The summary data tables of the PDG give the number of Eq. (3).

In this short note we show that such a value is grossly inconsistent with the total width of the $K_0^*(1430)$ and the saturation of this width with the $K\eta$ and $K\pi$ decay channels, with no trace of $K_0^*(1430) \rightarrow K_1(1270)\pi$ decay.

The quantum numbers of the $K_1(1270)$ are $I(J^P) = \frac{1}{2}(1^+)$ and for the scalar meson $K_0^*(1430)$ $\frac{1}{2}(0^+)$. The transition from $K_1(1270) \rightarrow K_0^*(1430)\pi$ with π $1(0^-)$ requires a p -wave coupling to conserve angular momentum and parity. This, together with the isospin coupling of a π to two isospin $\frac{1}{2}$ structures leads to the transition t -matrix from $K_1(1270) \rightarrow K_0^*(1430)\pi$

$$-it = C \epsilon^\mu p_{\pi\mu} \vec{\tau} \cdot \vec{\phi}, \quad (5)$$

with ϵ^μ the K_1 polarization vector, $\vec{\phi}$ the pion field in Cartesian basis and $\vec{\tau}$ the Pauli matrix acting on spinors of isospin $\frac{1}{2}$ ($\vec{\tau} \cdot \vec{\phi}$ gives $\sqrt{2}$ for $K_1^{(+)} \rightarrow \pi^+ K_0^{*(0)}$ and 1 for $K_1^{(+)} \rightarrow \pi^0 K_0^{*(+)}$).

The $K_1(1270) \rightarrow K_0^*(1430)\pi$ decay width is given by

$$\Gamma_1 = \frac{1}{8\pi} \frac{1}{M_{K_1}^2} p_\pi \overline{\sum} \sum |t|^2, \quad (6)$$

where $\overline{\sum} \sum |t|^2$ is the spin, isospin sum and average over the third components,

$$\overline{\sum} \sum_{J\text{-pol isos}} |t|^2 = 3 c^2 \frac{1}{3} \sum_{\text{pol}} \epsilon_\mu^* p_\pi^\mu \epsilon_\nu p_\pi^\nu, \quad (7)$$

and

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$$\sum_{\text{pol}} \epsilon_{\mu}^* \epsilon_{\nu} = -g_{\mu\nu} + \frac{P_{\mu} P_{\nu}}{M_{K_1}^2}, \quad (8)$$

with P^{μ} the K_1 momentum.

For the K_1 at rest one finds

$$\Gamma_1 = \frac{1}{8\pi} \frac{1}{M_{K_1}^2} C^2 p_{\pi}^3 B_1^2(p_{\pi}), \quad (9)$$

with

$$p_{\pi} = \frac{\lambda^{1/2}(M_{K_1}^2, M_{K_0^*}^2, m_{\pi}^2)}{2M_{K_1}} \theta(M_{K_1} - M_{K_0^*} - m_{\pi}), \quad (10)$$

where $\lambda(x, y, z) = (x + y - z)^2 - 4xy$ is the Källén function. In Eq. (9) we have also included a p-wave Blatt-Weisskopf barrier penetration factor [7] with the parametrization used in [8], $B_1(p) = 1/\sqrt{1 + (pR)^2}$, with R a range parameter given by $R = 0.25$ fm [8], curing an otherwise formal divergence of the integrals needed later on. Note that this factor helps softening the otherwise unrealistically large short distance behaviour.

Certainly Eq. (9) only makes sense if the widths of the K_1 and K_0^* are taken into account, and the overlap of their mass distributions allows the $K_1(1270)$ to have mass components bigger than the mass of the K_0^* plus a pion mass, something not easy given the mass of the $K_0^*(1430)$, but eased because of its large width. From the PDG we have

$$M_{K_0^*(1430)} = 1425 \pm 50 \text{ MeV}, \quad \Gamma_{K_0^*(1430)} = 270 \pm 80 \text{ MeV}. \quad (11)$$

To take into account the mass distributions of the two resonances we must convolve the width of Eq. (9) with the spectral functions of the resonances:

$$S_R(\tilde{M}) = \frac{-1}{\pi} \text{Im} \frac{1}{\tilde{M}^2 - M_R^2 + iM_R\Gamma_R}. \quad (12)$$

Then we have

$$\tilde{\Gamma}_1 = \frac{1}{N_1 N_2} \int d\tilde{M}_{K_1}^2 \times \int d\tilde{M}_{K_0^*}^2 S_{K_1}(\tilde{M}_{K_1}) \cdot S_{K_0^*}(\tilde{M}_{K_0^*}) \cdot \Gamma_1(\tilde{M}_{K_1}, \tilde{M}_{K_0^*}), \quad (13)$$

and N_1, N_2 are normalization factors used to account for some missing strength when integrating $d\tilde{M}_{K_1}, d\tilde{M}_{K_0^*}$ in some reasonable limits like $M_R \pm 2\Gamma_R$. We have

$$N_i = \int S_i(\tilde{M}_i) d\tilde{M}_i^2, \quad (i = K_1, K_0^*) \quad (14)$$

with the same limits for the integration as in Eq. (13). Note that the Blatt-Weisskopf factor, B_1 , in Eq. (9), acts as a natural regulator of the integral in Eq. (13) which would be otherwise logarithmically divergent.

On the other hand, we can use the same Eq. (5) to describe the $K_0^*(1430) \rightarrow K_1(1270)\pi$ decay, which is the time reversal reaction concerning the K_i states. In this case we evaluate $|t'|^2$ in the rest frame of the $K_0^*(1430)$ and we find

$$\overline{\sum \sum} |t'|^2 = 3C^2 \left[\frac{(M_{K_0^*}^2 - m_{\pi}^2 - M_{K_1}^2)^2}{4M_{K_1}^2} - m_{\pi}^2 \right] \quad (15)$$

and

$$\Gamma_{K_0^*} = \frac{1}{8\pi} \frac{1}{M_{K_0^*}^2} p'_{\pi} B_1^2(p'_{\pi}) \overline{\sum \sum} |t'|^2, \quad (16)$$

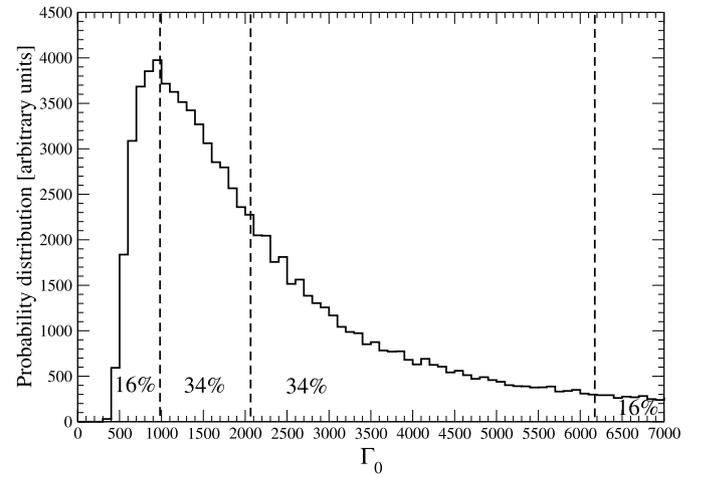


Fig. 1. Probability distribution obtained for the $K_0^*(1430) \rightarrow K_1(1270)\pi$ decay width, Γ_0 , by implementing a Monte Carlo sampling of the parameters within their errors.

with

$$p'_{\pi} = \frac{\lambda^{1/2}(M_{K_0^*}^2, M_{K_1}^2, m_{\pi}^2)}{2M_{K_0^*}} \theta(M_{K_0^*} - M_{K_1} - m_{\pi}). \quad (17)$$

Note also the inclusion of the Blatt-Weisskopf barrier penetration factor, B_1 . Once again we must use the convolution of Eq. (13) to obtain $\tilde{\Gamma}_{K_0^*}$ that takes into account the K_1 and K_0^* mass distributions.

If we take into account the nominal masses and widths of the K_1 and K_0^* of Eqs. (2) and (11) and the nominal value of the K_1 width to $K_0^*(1430)$ of Eq. (3) to obtain the value of the constant C , then we obtain

$$\Gamma_0 \equiv \Gamma_{K_0^* \rightarrow K_1 \pi^0} = 2082 \text{ MeV}. \quad (18)$$

This is a huge number, if not absurd, at odds with the total width of the K_0^* of 270 MeV. The contrast is even bigger when we see in the PDG that the width of the $K_0^*(1430)$ is practically exhausted with the $K\eta$ and $K\pi$ decays, and there is no experiment having reported the $K_0^*(1430) \rightarrow K_1(1270)\pi$ decay. We should note that even if we take the Belle results of Ref. [6] shown in Eq. (4), excluded “for averages, fits, limits, etc.” in the PDG, the $K_0^* \rightarrow K_1\pi$ decay width would be 170 MeV, smaller than the total K_0^* width, but still incompatible with the fact that the $K\eta$ and $K\pi$ decays practically exhaust the K_0^* decay width.

To further quantify the inconsistency of the PDG data on this partial decay width, Γ_0 , we carry out an error analysis taking into account all uncertainties of the different magnitudes. This error estimation is also called for since the $K_1 \rightarrow K_0^*\pi$ decay can proceed only from the overlapping of the spectral distributions in Eq. (13) and then slight differences in the values of the parameters affecting the spectral distributions can lead to large differences in our prediction of the final $K_0^* \rightarrow K_1\pi$ decay width. We perform a Monte Carlo sampling of the parameters in Eqs. (2), (3) and (11) within their errors and we find the probability distribution, $\rho(\Gamma_0)$, shown in Fig. 1.

We can see that the distribution of the $K_0^* \rightarrow K_1\pi$ decay width is very asymmetrical, implying a highly nonlinear dependence on the parameters. Indeed, for many values of the random generated parameters there is none or very little phase space allowed for the $K_1 \rightarrow K_0^*\pi$ decay, which makes the predicted $K_0^* \rightarrow K_1\pi$ width to be very large and then it moves much strength of the right tail of the probability distribution to high energies. Therefore we cannot

provide a Gaussian error but rather we can summarize the probability distribution by means of other statistical parameters like the *median* (the value with 50% probability to the left and 50% to the right) (represented by the medium dashed line in Fig. 1), which is about 2075 MeV and which essentially coincides with the value obtained in Eq. (18) with the central values of the parameters. We can roughly assign a lower and upper error to the previous value by considering the band of the width which encompasses 68% of the probability (see Fig. 1) and then we have

$$\Gamma_{K_0^* \rightarrow K_1 \pi^0} = 2075_{-1100}^{+4100} \text{ MeV.} \quad (19)$$

The large value obtained for the upper error is again a consequence of the small region of the parameter space where the K_1 is able to decay into $K_0^* \pi$. On the other hand the maximum of the distribution appears at about 1000 MeV, but is still incompatible with the experimental K_0^* decay width. In addition, if we evaluate the *expected value*, or *mean*, of the $K_0^* \rightarrow K_1 \pi$ width, Γ_0 , as $\bar{\Gamma}_0 = \int \Gamma_0 \rho(\Gamma_0) d\Gamma_0 / \int \rho(\Gamma_0) d\Gamma_0$, we get a much larger value, $\bar{\Gamma}_0 = 3728$ MeV, due to the large long tail at the right of the distribution as discussed above. If we use the Belle results of Eq. (4) instead of the value in (3) we would get values of about 7% of those quoted above, but again incompatible with the experimental total width of the $K_0^*(1430)$ of 270 MeV coming almost completely from $K\eta$ and $K\pi$ channels. The exact value of Γ_0 does not matter since we do not aim at providing an accurate value for it but to show the inconsistency of the $K_1(1270) \rightarrow \pi K_0^*(1430)$ quoted in the PDG.

On the other hand, we now recall that the PDG result of Eq. (3) was obtained from the work of Ref. [5]. The data of this work were reanalyzed in Ref. [9] to the light of the results of Ref. [10] in the study of the vector-pseudoscalar interaction with the chiral unitary approach, where two $K_1(1270)$ states were obtained coupling mostly to ρK and $K^* \pi$ respectively (see also the review paper [11]). The data of Ref. [5] clearly showed the ρK and $K^* \pi$ distributions peaking at different energies, but the analysis of Ref. [5], redone in Ref. [9], obtained these structures from subtle interference of the amplitudes used in their analysis, which were model dependent. It was shown in Ref. [9] that the peaks observed experimentally were well reproduced by the two $K_1(1270)$ states picture. The analysis of Ref. [5] also relied on the SU(3) mixture of the $K_1(1270)$ and the $K_1(1400)$ resonances that in Ref. [9] was discussed critically to the light of the existence of two $K_1(1270)$ states.

A revision and reanalysis of the data that led to the claim of the present PDG data for the $K_1(1270) \rightarrow K_0^*(1430)\pi$ partial decay width is necessary and the new results of the Belle Collaboration [6] seem to indicate that the official PDG results are grossly overcounted. Yet, we believe that a final answer to this question will require an analysis along the lines discussed in Ref. [9] for the $\rho K, K^* \pi$ decay modes, with the explicit consideration of the two $K_1(1270)$ states.

We think that it is important to pile up more experimental information supporting the existence of two $K_1(1270)$ states, and

take advantage to recall the suggestions made in the literature to attain this goal using the different reactions:

- 1) $\tau \rightarrow \nu_\tau P^- K_1(1270)$, with $P^- \equiv \pi^-, K^-$ [12];
- 2) $D^0 \rightarrow \pi^+ V P$, with $V P \equiv \rho K, K^* \pi$ [13];
- 3) $D^+ \rightarrow \nu e^+ V P$, with $V P \equiv \rho K, K^* \pi$ [14];
- 4) τ decay to ν_τ and two $K_1(1270)$ states [15];
- 5) $\bar{B} \rightarrow J/\psi V P$, with $V P \equiv \rho \bar{K}, \bar{K}^* \pi$ [16].

These and other reactions where $K\pi\pi$ is obtained in the final states, separating the ρK and $K^* \pi$ modes, will be most useful in the future to settle the issue of the two poles of the $K_1(1270)$ and at the same time resolve the problem of the flagrant inconsistency of the present PDG data on the $K_1(1270) \rightarrow K_0^*(1430)\pi$ decay.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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