A Discrete Competitive Facility Location Model with Proportional and Binary Rules Sequentially Applied

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Abstract

The paper is focused on discrete competitive facility location problem for an entering firm considering different customer behavior models: for essential goods, customers generally spread their buying power among all facilities within an attraction area, but if there are no facilities nearby, then customers choose a single highly attractive facility outside the attraction area to satisfy their demand. The new facility location model has been proposed considering the proportional customer choice rule for customers with facilities within the attraction area and the binary rule – for customers which facilities are located outside the attraction area. The model has been formulated as a non-linear binary programming problem and a heuristic optimization algorithm has been applied to find the optimal solutions for different instances of the problem using real geographical coordinates and population data of thousands of municipalities in Spain.

Keywords: Location, Competitive model, Proportional and binary customers rules, Discrete optimization, Random search

1 Introduction

In a market environment, competing facilities provide similar products or services for customers' patronage, for instance, essential goods. A competitive facility location model tries to find the best locations for some new facilities. The goal of the competitive facility location problem is to maximize the market share captured by these facilities (see survey papers [1–3]). These models can be classified according to their competition type, location space and customer behavior.

Competition can be static, with foresight, or dynamic. In a case of static competition, there are some pre-existing facilities owned by competitors and they don't react when a new firm enters in the market [4]. In the second case, potential competitors are not yet on the market but will be present shortly after the new facilities are located (the leader locates some facilities to maximize their total market share after the follower locates their facilities [5]). In the last case firms repeatedly re-optimize their locations [6].

The location space can be discrete (locations for new facilities are chosen from a discrete set of candidates [7]), a network (where facilities can be located at network nodes or at inner edge points [8]), or the plane (the new facilities can be located at any point in a certain region [9]).

Customer behavior plays a very important role in competitive location models. Different customer behaviors can be described in a precise manner with different rules, which are often expressed as attraction functions that customers feel from facilities. The most common customer choice rules are the ones called proportional and binary [10]. Following the proportional rule the customers patronize all the facilities in proportion to facility attraction (see for instance [9, 11, 12]). In the case of binary rule the customer patronizes the most attractive facility [7, 13, 14]. These rules can describe most of the customer's behavior in competitive location models, but sometimes some variations or combinations of them are necessary for a better fit of the model.

In this paper we present a discrete competitive location model in which the proportional and binary rules will be considered simultaneously, but their application will be sequential when the customer demand is split between operating facilities, i.e., the proportional rule will be the main one and will be the first to be applied, and the binary rule will be the secondary one. The binary rule will only be applied when it is not possible to apply the proportional rule for the distribution of customer demand.

Once all facilities have been located, pre-existing and new, for each customer it is checked whether there are located facilities that have at least a minimum attraction threshold that the customer has pre-established in advance. If such facilities exist, their demand is distributed among them in proportion to the attraction that the customer feels for each of them (proportional rule). If there are no such facilities, that is, if for a customer there are no facilities located with a minimum attractiveness, since they are essential goods, the customer will satisfy his demand for the facility or facilities with the greatest attraction (binary rule). If there are several facilities with maximum attraction (all with insufficient attraction for the proportional rule), the demand of this customer is equally distributed among all of them, whether they are pre-existing or new facilities [13].

The reminder of the paper is organized as follows: Section 2 consists of the description of the model and its formulation as a binary nonlinear programming problem, Section 3 includes an illustrative example for a better understanding of the model, Section 4 is devoted to the numerical experiment by applying heuristic algorithm to search for an optimal solution, and conclusions are formulated in Section 5.

2 The model

An entering firm wants to open its facilities in a market area where other competing firms are already operating. For simplicity all these preexisting facilities will be considered as a single competitor. In order to maximize the market share captured, the entering firm must decide where to locate its new s facilities among a discrete set of possible locations. We assume that customers are concentrated in some demand points, that their demand is fixed and known, and that the products are essential, so each customer's demand has to be satisfied.

An important issue in this type of models is to know how customers select the facilities that will serve their demand (customer behavior). If we study the behavior of the inhabitants of a population when acquiring essential goods (food, drink, sanitary products, cleaning supplies, etc.), we can observe that they are usually acquired in different establishments within an area, for what the buying power of customers is distributed among all of them in proportion to the attraction that the customer feels for them. This area of influence, in the simplest cases, can only be determined by the distance that customers are willing to travel to acquire the products, but in general, and if we consider the characteristics of the facilities when determining the latter area (their quality), the facilities within this area will be those with a minimum attraction value for customers. But there are situations in which there are no sufficiently attractive establishments for the customer to purchase these goods in his area of influence. In these cases, and because they usually are associated with a greater travel distance, they tend to choose a single facility where they can purchase all the products and that offers a large number of services that make it more attractive to the customer. This facility will be the most attractive among all the located facilities, belonging to the incoming firm or to the competitor. This type of situation is what we try to model in this paper.

In this paper we are going to consider two customer choice rules, but unlike other papers in the literature, in this case we are going to use both rules sequentially, that is, first we will apply the proportional rule to all customers for which there are facilities with a minimum attraction threshold fixed by each of them, pre-existing or new ones, and then, and only for customers for whom the proportional rule cannot be applied, we will apply the binary rule to identify the facility or facilities that will serve their demand.

2.1 Notation

The following general notation is used:

Indices

i, I	index and set of demand points (customers)
j	index of facilities

Data

w_i	demand of customers at demand point i.
q_j	quality of facility j.
d_{ij}	distance between demand point i and facility j
a_{ij}	attraction that customers at demand point i feel for facility j, $a_{ij} = \frac{q_j}{1+d_{ij}}$
A_i	minimum attraction required by customer i for a facility to serve its demand
F	pre-existing facilities of competitors
$a_i(F)$	maximum attraction that customers at demand point i feel for facilities in F, $a_i(F) = \max\{a_{ij} : j \in F\}$
L	a set of possible locations for the new facilities.

Variables

X	set of new facilities locations, $X \in L, \mathsf{X} = s$.

Let $I_0 = \{i \in I : \exists j \in F$ such that $a_{ij} \geq A_i\}$ be customers whose demand will be certainly served by using the proportional rule, it does not matter where the new facilities are located, because before the entering firm locates their facilities, there already are some pre-existing facilities with attraction greater that the minimum attraction threshold required by these customers. Let $I_1 = I \backslash I_0$ be customers that are captured by competitors if the binary rule is used, and whose demand can be captured by the entering firm if any facility with attraction at least the minimum attraction threshold or the maximum attraction of competitors is located, and could be fully or partially captured.

Given $i \in I_1$, it follows that $a_i(F) < A_i$, then:

- If $a_{ij} < a_i(F), \forall j \in X$, the entering firm does not capture its demand.
- If $a_{ij} > a_i(F)$, for any $j \in X$, the entering firm captures its full demand, regardless of whether the proportional or binary rule is used.
- If $a_i(X) = a_i(F)$, the binary rule is used and its demand will be served equally by all facilities with its maximum attraction.

For each $i \in I$, set $L_i^{>} = \{j \in L : a_{ij} > a_i(F)\}$ and $L_i^{=} = \{j \in L : a_{ij} = a_i(F)\}$, then the following sets are considered:

$I_1^1 = \{i \in I_1 : L_i^{>} \neq \emptyset\}$ and $I_1^2 = \{i \in I_1 : L_i^{=} \neq \emptyset\}$

where I_1^1 are customers of I_1 which demand could be fully captured by the entering firm, and I_1^2 are customers of I_1 which demand, if captured by the entering firm, will be partially (see Figure 1). Note that I_1^1 and I_1^2 do not have to be disjoint.

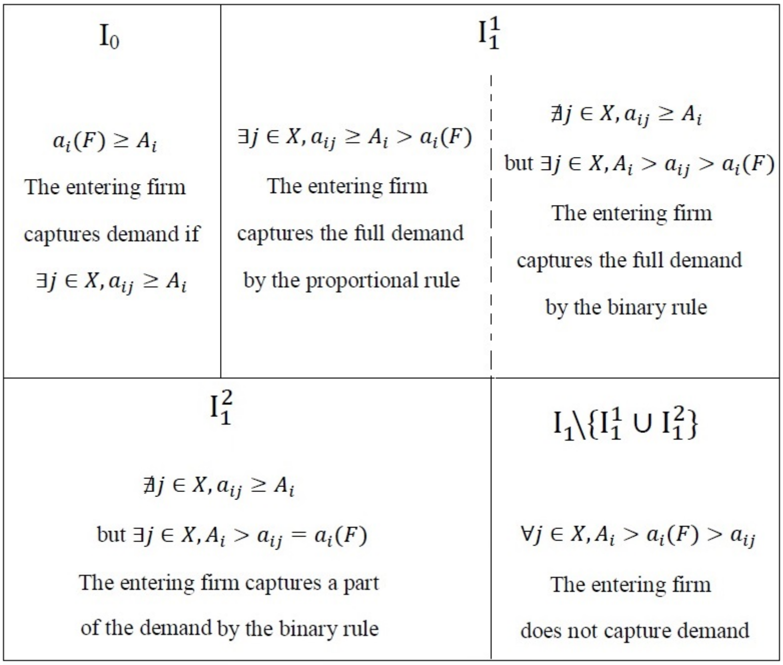

Fig. 1 Different subsets of I

In addition, for each $i \in I_0$, it is necessary to know the facilities from which their demand could be served with the proportional rule, whether they are pre-existing or location candidates for the new facilities, so the following sets are defined:

$L_i^p = \{j \in L : a_{ij} \geq A_i\}$ and $F_i^p = \{j \in F : a_{ij} \geq A_i\}$

2.2 Formulation

Consider the following variables:

$$x_j = \begin{cases} 1 \text{ if a new facility is located at } j \\ 0 \text{ otherwise} \end{cases} \qquad j \in L$$

$$z_i = \begin{cases} 1 \text{ if the demand of } i \text{ is full captured} \\ \quad \text{by the entering firm} \\ 0 \text{ otherwise} \end{cases} \qquad i \in I_1^1$$

$$z_{ij} = \begin{cases} 1 \text{ if the demand of } i \text{ is partially captured by } j \\ 0 \text{ otherwise} \end{cases} \qquad i \in I_1^2, j \in L_i^=$$

Then the basic model has the following formulation as a binary nonlinear programming problem:

$$(P) \begin{cases} \text{Max } \sum_{i \in I_0} \frac{\sum_{j \in L_i^p} a_{ij} x_j}{\sum_{j \in L_i^p} a_{ij} x_j + \sum_{j \in F_i^p} a_{ij}} w_i + \sum_{i \in I_1^1} z_i w_i + \\ \quad + \sum_{i \in I_1^2} \frac{\sum_{j \in L_i^=} z_{ij}}{\sum_{j \in L_i^=} z_{ij} + n_i(F)} w_i \\ \text{s.t.} \quad \sum_{j \in L} x_j = s & (1) \\ \quad z_i \leq \sum_{j \in L_i^>} x_j, \forall i \in I_1^1 & (2) \\ \quad z_{ij} \leq x_j, \forall i \in I_1^2, \forall j \in L_i^= & (3) \\ \quad \sum_{j \in L_i^=} z_{ij} \leq (1 - z_i)|L_i^=|, \forall i \in I_1^2 & (4) \\ \quad x_j \in \{0,1\}, \forall j \in L & (5) \\ \quad z_i \in \{0,1\}, \forall i \in I_1^1 & (6) \\ \quad z_{ij} \in \{0,1\}, \forall i \in I_1^2, \forall j \in L_i^= & (7) \end{cases}$$

where $n_i(F) = |\{j \in F : a_{ij} = a_i(F)\}|$.

The first term of the objective function is the captured demand from customers at I_0, who use the proportional rule to divide their buying power. The second term is related to the customer demand in I_1 captured completely by the entering firm, no matter the used rule. And the third term refers to the customer demand captured by using the binary rule, which is equally divided among all the facilities with the maximum attraction for these customers. Constraint (1) is the number of new facilities to be located by the entering firm. Constraint set (2) states that a customer's demand at I_1^1 will be fully captured if any facility is located with a higher attraction than the competitors. Constraint sets (3) and (4) set that to partially capture demand using the binary rule, facilities with the same maximum attraction than the competitors must be located, and their number is limited by the number of location candidates that verify this condition. The rest of constraint sets are the binary condition for the variables.

3 An Illustrative Example

For simplicity, the same quality has been assumed for all facilities, existing and new, then $q_j = 1, \forall j \in F \cup L$. Then, since

$$a_{ij} = \frac{q_j}{1 + d_{ij}}, \forall i \in I, j \in F \cup L, \tag{8}$$

the minimum threshold of attraction for each customer i, A_i, is equivalent to a maximum distance between customers and facilities that will serve their demand. In this case, each customer has associated a fixed distance radius R_i where they can spend their demand power. The higher the minimum attraction threshold, the smaller the radius of distance. If there are facilities located within this radius, the proportional rule can be applied and their demand will be served by all these facilities in proportion to their attractiveness (inversely proportional to distance). If there are no facilities located within this radius, then the binary rule should be applied and their demand will be divided equally among all facilities most attractive to this customer (the closest).

Figure 2 illustrates the model with 10 demand points each one have different minimum attraction threshold (i.e., a different distance radius), where the pre-existing facilities owned to competitors are squares ($F = \{P, Q\}$) and the location candidates for the entering firm are triangles ($L = \{A, B, C, D, E, F\}$). Suppose that $s = 3$ new facilities are being located which optimal locations are marked by solid triangles ($X = \{B, D, E\}$). Solid lines denote the proportional rule, and dashed lines – the binary rule.

Before locating new facilities we know that $I_0 = \{2, 3, 5, 7, 9\}$, $I_1^1 = \{1, 6, 8\}$, $I_1^2 = \{10\}$, and customer 4 will never be captured by an entering firm since there is no location candidates within its radius while the closest location outside the radius is occupied by the preexisting facility.

After locating new facilities in their optimal locations (filled triangles in the figure), the demand of customers in demand point 2 is fully captured by competitors, the demand of customers in 8 is fully captured by the entering firm, while the demand of customers in 3, 5, 7 and 9 are split between several facilities owned by competitors and entering firm following the proportional rule. Customers in demand points 1, 4, 6, and 10 do not have any facility located within their distance radius and binary rule is applied to serve their demand by the closest facility outside the threshold radius: customers in demand points 1 and 6 are served by new facilities, customer in 4 – by a competitors' facility. Customers in 10 split their buying power equally among two facilities since both are at the same distance.

4 Numerical Experiments

The formulated facility location problem has been solved by Genetic Algorithm (GA), which has proven to be efficient in solving this type of problems [15], using geographical coordinates and population of municipalities in Spain as

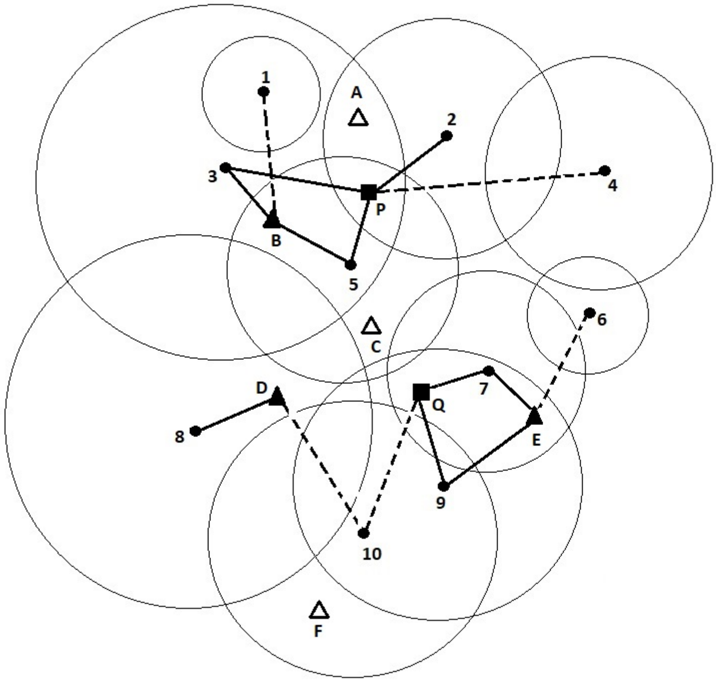

Fig. 2 Illustrative example of the proposed model.

demand points. The distances between demand points and facilities have been calculated in kilometers using great circle principle – Haversine distance [16]. Preexisting facilities were located in 10 most populated demand points ($F = \{1, 2, \ldots, 10\}$). The set of 100 candidate locations for the new facilities has been formed of 100 most populated demand points including those where preexisting facilities already located ($L = \{1, 2, \ldots, 100\}$). It was expected to find optimal locations for 3 new facilities.

Equal qualities were set for all preexisting and new facilities ($q_j = 1$, $\forall j \in F \cup L$) in order to focus on the distance factor only when evaluating attraction of a facility. Four different attraction values $A_i \in \{0.01, 0.02, 0.05, 0.1\}$ as threshold of changing customer behavior rule have been considered for all customers, which is equivalent to associate a radius of 99, 49, 19 and 9 kilometers to each customer, respectively. If there are facilities located within this radius, the proportional rule can be applied and the demand will be served by all these facilities in proportion to their attractiveness. If there are no facilities located within this radius, then the binary rule will be applied and the demand will be divided equally among all facilities most attractive to this customer.

GA has been applied for the latter discrete facility location problem using strategies presented in [17]. Population size of 100 individuals with probability equal to 0.8 for uniform crossover and probability equal to 1 divided by the number of new facilities for mutation were used. One hundred generations of

Table 1 Results obtained solving CFLP with proportional and binary customer behavior rules and different threshold values.

Threshold	Total	Proportional	Binary
0.01	30.85 ($\pm$0.013)	20.81	10.04
0.02	30.83 ($\pm$0.007)	15.34	15.49
0.05	31.52 ($\pm$0.008)	11.13	20.39
0.10	30.79 ($\pm$0.017)	7.49	23.30

the algorithm were performed thus devoting 10,000 function evaluations to approximate the optimal solution of a single instance of the problem. Due to stochastic nature of the algorithm 100 independent runs were performed and statistical estimates were calculated. Average results obtained for different threshold values are presented in Table 1, where the first column stands for the threshold value, the second – for the average total market share obtained by the new facilities with standard deviation, and the last two columns – for the average market share obtained by the proportional and the binary rules respectively. All results are expressed in percents from the total markets share available in the geographical area described by the given data set.

One can see from the table, the total market share of the new facilities is about 31% from the total market share value with standard deviation varying from 0.007 to 0.017. When the threshold values is 0.01 ($\sim$100 km), two thirds of the total market share were obtained following the proportional customer behavior rule while the remaining third was obtained by the binary customer behavior rule. When the threshold values is increased to 0.02 ($\sim$50 km), then almost the same market share of the new facilities are attracted almost equally by the proportional and the binary customer behavior rule. Further increment of the threshold value makes the binary rule dominant against the proportional customer behavior rule.

5 Conclusions and Future Work

A new discrete competitive location model has been introduced in this paper. As the main novelty of the model, two customer choice rules have been used sequentially: the proportional rule as the main one and the binary rule as the secondary one. For this, each customer has a minimum attraction threshold so that the proportional rule can be applied to satisfy his demand. After locating the facilities of the entering firm, if a customer has facilities that satisfy his minimum attraction threshold, pre-existing or new, then the proportional rule is applied to distribute his demand among them in proportion to his attraction, but if there are none, then the binary choice rule is used to determine which facility or facilities (if there are ties in the maximum attraction) will serve his demand.

The first formulation of the model has been proposed as a nonlinear binary programming problem, where different sets of customers and variables have been defined in order to model the sequential use of the two customer choice rules applied in the model. Due to the fact that the model is non-linear, a heuristic algorithm has been applied to approximate the optimal solution for the problem which uses real data of population and geographical coordinates of the municipalities of Spain.

Our future work will focus on improvement of the ingredients of the proposed model, using different and more realistic customer choice rules as well as considering probabilistic methods to evaluate attractions of the facilities. Also, we will try to linearize the improved model, and apply the previously proposed algorithm for discrete facility location which is based on ranking-based of the candidate locations and investigate the dependence of the distribution of pre-existing facilities among the geographical area on the customer behavior and complexity of the optimization problem.

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References

[1] Drezner, T. (2014). A review of competitive facility location in the plane. *Logistic Research* 7, 114.

[2] Eiselt, H., Marianov, V., Drezner, T. (2015). Competitive location models. In G. Laporte, S. Nickel, F.S. da Gama (Eds.), *Location Science*, 365–398.

[3] ReVelle, C., Eiselt, H., Daskin, M. (2008). A bibliography for some fundamental problem categories in discrete location science. *European Journal of Operational Research* 184(3), 817–48.

[4] Plastria, F. (2001). Static competitive facility location: an overview of optimisation approaches. *European Journal of Operational Research* 129, 461–470.

[5] Plastria, F., Vanhaverbeke, L. (2008). Discrete models for competitive location with foresight. *Computers and Operations Research* 35, 683–700.

[6] Farahani, R.Z., Abedian, M., Sharahi, S. (2009). In Farahani R.Z. and Hekmatfar M. Eds. *Facility location: concepts, models, algorithms and case studies.* Springer-Verlag, 347–372.

[7] Fernández, P., Pelegrín, B., Lančinskas, A., Žilinskas J. (2017). New heuristic algorithms for discrete competitive location problems with binary and partially binary customer behavior. *Computers and Operations Research* 79, 12–18.

[8] Grohmann, S., Urošević, D., Carrizosa, E., Mladenović, N. (2017). Solving multifacility huff location models on networks using metaheuristic and exact approaches. *Computers and Operations Research* 78, 537–546.

[9] Fernández, J., G.-Tóth, B., Redondo, J.L., Ortigosa, P.M. (2019) The probabilistic customer's choice rule with a threshold attraction value: effect on the location of competitive facilities in the plane. *Computers and Operations Research* 101, 234–249.

[10] Serra, D., Eiselt, H.A., Laporte, G., ReVelle, C.S. (1999). Market capture models under various customer-choice rules. *Environment and Planning B: Planning and Design* 26, 741–750.

[11] Fernández, P., Pelegrín, B., Lančinskas, A. Žilinskas, J. (2021). Exact and heuristic solutions of a discrete competitive location model with Pareto-Huff customer choice rule. *Journal of Computational and Applied Mathematics*, 385, 113200.

[12] Drezner, T., Drezner, Z. (2004). Finding the optimal solution to the Huff based competitive location model. *Computational Management Science* 1(2), 193–208.

[13] Pelegrín, B., Fernández, P.,García, M.D. (2015). On tie breaking in competitive location under binary customer behavior. *OMEGA-International Journal of Management Science* 52, 156–167.

[14] Campos, C.M., Santos-Peñate, D.R., Moreno, J.A. (2010). An exact procedure and LP formulations for the leader-follower location problem. *Top* 18(1), 97–121.

[15] Pelegrín, B., Redondo, J.L., Fernández, P. et al. (2007). GASUB: finding global optima to discrete location problems by a genetic-like algorithm. *Journal of Global Optimization* 38, 249—264.

[16] Sinnott, R.W. (1984). Virtues of the haversine. *Sky & Telescope* 68(2), 158.

[17] Lančinskas, A., Pascual, F., Pelegrín B., ReVelle, C.S. (2017). Improving solution of discrete competitive facility location problems. *Optimization Letters* 11(2), 259–270.