Collusion and Customization*

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December 18, 2019

Abstract

We analyze the effect of customizing a product on the ability of firms to tacitly collude on prices. Following Bar-Isaac et al. (2014), we allow firms to be located inside the circle in the Salop model (1979). Our analysis shows that the effect of product customization on the stability of collusion depends on the sensitivity of consumers' utility to the degree of customization. In particular, if that sensitivity is low enough then greater customization facilitates collusion. Otherwise, greater customization hinders collusion if consumers value the product little.

Keywords: Collusion; Customization; The Salop model

JEL classification: D40; L10; L40

*I thank the comments of the editor, the referee and the participants at the XIX Encuentro de Economía Aplicada (Sevilla), XXXI Jornadas de Economía Industrial (Mallorca) and 41 Simposio de la Asociación Española de Economía (Bilbao). I acknowledge financial support from the Spanish Ministro de Ciencia, Innovación y Universidades and Spanish Agencia Estatal de Investigación, under projects ECO2016-76178-P and PID2019-107192GB-I00, which are co-financed by FEDER funds. Any remaining errors are mine alone.

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1 Introduction

Greater customization of a product makes it more useful for consumers who value the product more, but less customization makes it more useful for consumers who value the product less and less useful for those who value it more. Changes in the product design lead to rotations in demand according to Johnson and Myatt (2006). These rotations in demand may affect the ability of firms to tacitly collude on prices. This paper therefore sets out to investigate the effect of customizing a product on the ability of firms to tacitly collude on prices. To that end, we use the version of the Salop circle model (1979) developed by Bar-Isaac et al. (2014), which allows firms to be located inside the circle. Thus, a product is characterized by two dimensions: a horizontal dimension that reflects the variety of a product and a vertical dimension that reflects the degree of customization of that product. Under this framework, Bar-Isaac et al. (2014) analyze three models: monopoly, duopoly and a model of monopolistic competition in which consumers incur search costs to observe products. They provide the sufficient conditions that ensure extreme or intermediate design, and show that firms with higher marginal costs choose more targeted designs.[1]

One industry in which this model can be applied is the media industry. For example, the newspapers industry. In this case, a newspaper might choose a diverse and varied edition aimed at the general public, or a specific edition focused primarily on a specific audience (for instance, sports fans). Notice that if the newspaper increases the information in sporting events, the utility of sports fans increases, but the utility of other consumers decreases. Otherwise, if the newspaper reduces information about sporting events and increases information about the general public, the utility of sports fans decreases, but the utility of other consumers increases. Moreover, there is evidence that some firms are colluding in the newspapers market. In particular, Argentesi and Filistrucchi (2007) provide evidence that newspapers in Italy have been colluding on the cover price but not on the advertising tariffs.

Although the literature has not previously analyzed collusion under the framework considered by Bar-Isaac et al. (2014), collusion in markets for horizontally differentiated products has been analyzed by Chang (1991). He develops a model à la Hotelling (1929) in which firms play trigger strategies as in Friedman (1971). His principal finding is that the smaller the degree of product differentiation is, the harder firms find it to collude. Moreover, Häckner (1996) has shown that Chang's results are robust to changes in the mechanism of punishment for deviating from collusion. On the other hand, Häckner (1994) analyzes collusion in markets for vertically differentiated products. He finds that collusion is more easily sustained the more similar the products are, which contrasts with the results obtained in horizontal product differentiation models (Chang (1991), Häckner (1996)).

Our paper shows that the effect of customizing a product on the stability of collusion depends on the sensitivity of consumers' utility to the degree

[1] González-Maestre and Granero (2018) extend the model of Bar-Isaac et al. (2014) to focus on the analysis of strategic pricing, variety and welfare.

of customization. In particular, if that sensitivity is low enough then greater customization facilitates collusion. Otherwise, greater customization hinders collusion if consumers value the product little. We also provide a welfare analysis. Under collusion, which is modeled as a multiproduct monopoly, it is found that the effects of customizing depend on the sensitivity of consumers' utility to the degree of customization, but the effect on the consumer surplus is always positive. Thus, if that sensitivity is low enough a more customized product entails a lower price, profit and welfare. Otherwise, the opposite result is obtained. On the other hand, at the stage of punishment, which is modeled as a duopoly, it is obtained that a customized product implies a higher price, which in turn leads to a higher profit. However, the effect of customizing on consumer surplus and welfare is negative if the sensitivity of consumers' utility to the degree of customization is low enough, but positive if it is high enough.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 obtains the static equilibriums, and Section 4 obtains and analyzes the equilibrium in the supergame that we consider. Section 5 concludes.

2 The Model

There are two products, 1 and 2, which are modeled according to the version of the Salop (1979) circular model proposed by Bar-Isaac et al. (2014). Thus, products can locate not only on the circumference of a circle of radius 1 but also inside it. This means that a product is characterized by two dimensions: a horizontal dimension that refers to its variety, and a vertical dimension that refers to its degree of customization. We assume that products are located opposite each other. Without loss of generality we consider that product 1 is located at angle 0 and product 2 at angle 1/2. We assume that a unit mass of consumers is uniformly distributed around the circumference of a circle of radius 1. See Figure 1. It is assumed that each consumer can buy at most one unit of the product. Thus, the utility of a consumer located at x is:

$$U(x) = \begin{cases} v - C\left(1 - s_1\right) - ts_1 x - p_1 & \text{if he/she buys from firm 1,} \\ v - C\left(1 - s_2\right) - ts_2\left(\frac{1}{2} - x\right) - p_2 & \text{if he/she buys from firm 2,} \end{cases} \tag{1}$$

where v represents the consumer's utility obtained from buying the fully targeted design of her/his preferred product, $C\left(1 - s_i\right)$ $i = 1, 2$ is the vertical cost, $ts_1 x$ and $ts_2\left(1/2 - x\right)$ are the horizontal cost and p_i represents the price of the product $i = 1, 2$. We use the taxonomy of Bar-Isaac et al. (2014), so the horizontal cost is the disutility associated with consuming a different product (or variety), while the vertical cost represents the disutility associated with consuming a less customized product. From Figure 1 notice that a more customized product reduces the disutility from consuming a less customized product (vertical cost), but increases the horizontal cost from consuming a different product (or variety) because the travel along the arc is larger.

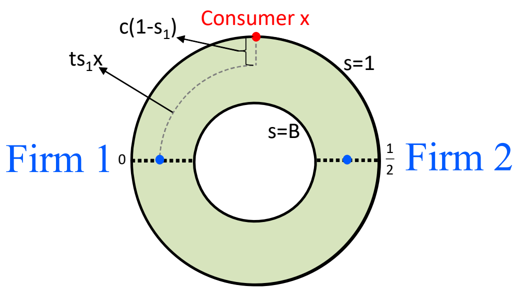

Figure 1: Firms and consumers.

Following Bar-Isaac et al. (2014), we assume that $C(.)$ is twice continuously differentiable and $C'(.) > 0$. This last assumption means that a more customized product increases consumer utility. Thus, $s_i \in [B, 1]$ represents the degree of customization of product $i = 1, 2$, so $s_i = 1$ indicates that the design of product i is fully customized and $s_i = B > 0$ that the design is as generic as possible. It is assumed that v is high enough for all consumers to buy at least from one firm. In particular, we assume the following:

Assumption 1 $v \geq C(1-s) + \frac{3}{4}st$.

From the utility function (1), it is possible to find the consumer who is indifferent between buying product 1 and product 2, which is given by:

$$\widehat{x}(p_1, p_2) = \frac{2C(1-s_2) - 2C(1-s_1) + ts_2 + 2p_2 - 2p_1}{2t(s_1 + s_2)}. \tag{2}$$

The demand functions of the two products are:

$$D_1(p_1, p_2) = 2\widehat{x} \text{ and } D_2(p_1, p_2) = 1 - D_1(p_1, p_2) \tag{3}$$

It is assumed that the fixed cost of developing a product and the marginal cost of production are zero. Thus, the profit function of each firm is $\pi_i(p_1, p_2) = p_i D_i(p_1, p_2)$ $i = 1, 2$.

Following Friedman (1971), we consider an infinitely repeated game in which firms play trigger strategies. In particular, firms start by charging collusive prices and continue charging those prices if neither firm has deviated in a previous stage. However, if either firm deviates at any stage then both firms revert to the Nash equilibrium at duopoly in the following stages. We assume perfect monitoring, so if a firm has deviated it is immediately detected but the punishment is implemented in the following stage.

We seek to find the subgame perfect equilibrium (SPE) of the infinitely repeated game. Thus, collusion on prices is an SPE of the game if and only if the present value of collusion profits exceeds the deviation profit plus the present value of the punishment profits of each firm, i.e. if and only if

$$\sum_{t=0}^{\infty}\delta^t\pi_i^C \geq \pi_i^D + \sum_{t=1}^{\infty}\delta^t\pi_i^N \ \forall i = 1, 2, \tag{4}$$

where δ represents the discount factor and π_i^C, π_i^D and π_i^N are the one period collusion, deviation and Nash profits of firm $i = 1, 2$, respectively. Given that we look for symmetric equilibrium, we assume that the designs of the two products are identical. Thus, the vertical cost is irrelevant to determine the demand for each product because it is the same for both products. Moreover, to make the paper more readable we eliminate subscript i on equilibrium prices and profits.

Assumption 2 *The design of the two products is identical, i.e. $s_1 = s_2 = s$.*

In the next section we look for the one period Nash equilibrium in duopoly and multiproduct monopoly, and the firms' optimal deviation strategies from the collusion agreement.

3 Analysis and results

3.1 Duopoly

In this subsection we solve a duopoly game, which represent the punishment stage if either firm deviates from collusive pricing. The timing of this game is as follows. First, firms simultaneously set prices. Next, consumers make their purchase decision. Substituting the demand functions (3) in the profit function gives:

$$\pi_i(p_1, p_2) = p_i \frac{ts + 2(p_j - p_i)}{2ts}, \ i = 1, 2 \ i \neq j$$

From the first order conditions of profit maximization, the following reaction functions by firms are obtained:[2]

$$p_i(p_j) = \frac{p_j}{2} + \frac{ts}{4}, \ i = 1, 2 \ i \neq j \tag{5}$$

From the intersection of the price reaction functions of the two firms the equilibrium prices can be found, and then the demands and profits, which are:

$$p^N = \frac{st}{2}; D^N = \frac{1}{2}; \pi^N = \frac{st}{4}.$$

The consumer surplus is defined as:

[2] The second order condition of the optimization problem is satisfied $\frac{\partial^2 \pi_i(p_1,p_2)}{\partial^2 p_i} = \frac{-4}{t(s_i+s_j)} < 0$.

$$CS = 2\left(\int_0^{\widehat{x}} u_1 dx + \int_{\widehat{x}}^{\frac{1}{2}} u_2 dx\right).$$

Welfare is defined as $W = CS + \pi_1 + \pi_2$. Therefore, the consumer surplus and welfare at equilibrium are:

$$CS^N = v - C(1-s) - \frac{5st}{8}; W^N = v - C(1-s) - \frac{st}{8}.$$

Notice that a more customized product decreases the vertical cost, which is the same for both products and is independent of the location of the consumers. Thus, the vertical cost is irrelevant to determine the demand for each product. However, a more customized product increases the horizontal cost from consuming a different product. Thus, it is more expensive for a consumer to buy from the rival when s increases. It implies greater market power for firms, so they set a higher price and obtain greater profits since the market is covered. As a result, a more customized product is more expensive.

On the other hand, the effect of the design of a product on consumer surplus and welfare is ambiguous. In particular, $CS^N$and W^N positively depend on s if $C'(1-s)$ is high enough. This is because the compensation between the two opposite effects of the product design depends on the magnitude of $C'(1-s)$. If $C'(1-s)$ is low enough, CS^N and W^N negatively depend on s, but they positively depend on s if $C'(1-s)$ is high enough. For intermediate values of $C'(1-s)$, W^N positively depend on s and CS^N negatively depends on s. These properties at the equilibrium of the duopoly game are summarized in the following proposition:

Proposition 1 *At the equilibrium of the duopoly game, the following is found:*

a) $\frac{\partial p^N}{\partial s} = \frac{t}{2} > 0$, $\frac{\partial \pi^N}{\partial s} = \frac{t}{4} > 0$.

b) $\frac{\partial CS^N}{\partial s} = C'(1-s) - \frac{5}{8}t > 0 \leftrightarrow C'(1-s) > \frac{5}{8}t$.

c) $\frac{\partial W^N}{\partial s} = C'(1-s) - \frac{t}{8} > 0 \leftrightarrow C'(1-s) > \frac{t}{8}$.

If $C'(1-s) < t/8$, both CS^N and W^N negatively depend on s. This is because the increase in the horizontal cost is not offset by the reduction in the vertical cost when s increases. However, if $C'(1-s) > 5t/8$, both CS^N and W^N positively depend on s because of the increase in the horizontal cost is offset by the reduction in the vertical cost. Finally, if $C'(1-s)$ takes intermediate values ($t/8 < C'(1-s) < 5t/8$), W^N positively depend on s, but CS^N negatively depends on s, because welfare also depends on firms' profits and the effect of s on profits is always positive. Thus, from a welfare point of view, the increase in horizontal cost is offset by the reduction in vertical cost, but not from the point of view of consumers.

3.2 Collusion

We now look for the equilibrium at the cooperative stage, in which firms collude on prices and behave as a multiproduct monopoly. Given that firms are symmetrical and are located opposite each other, they maximize their joint profits by raising prices until consumers with preferences $x = 1/4$ and $x = 3/4$ are indifferent between buying and not buying. Thus the prices, demands and profits are as follows:

Proposition 2 *At the equilibrium of the multiproduct monopoly the prices, profits, demands, consumer surplus and welfare are:*

$$\begin{aligned} p^C &= v - \frac{4C(1-s)+st}{4}; \pi^C = \frac{4v-4C(1-s)-st}{8} \\ D^C &= \frac{1}{2}; CS^C = \frac{st}{8}; W^C = \frac{8v-8C(1-s)-st}{8} \end{aligned} \tag{6}$$

Proof: see Appendix.

Unlike the duopoly, the effect of product design on prices is ambiguous. In particular, a more customized product implies a higher price if $C'(1-s)$ is high enough. When $C'(1-s)$ is high enough, the reduction in the disutility from consuming a less customized product is greater, so that it offsets the increase in the horizontal cost from consuming a different product even for those consumers located further. Thus, the price increases. Otherwise the price decreases. However, a more customized product increases the consumer surplus, even when the product becomes more expensive. Therefore, the effect of product design on welfare also depends on the marginal disutility from consuming a less customized product, $C'(1-s)$. These properties at the equilibrium are summarized in Proposition 3:

Proposition 3 *At the equilibrium of the multiproduct monopoly game, the following is found:*

a) $\frac{\partial p^C}{\partial s} = C'(1-s) - \frac{t}{4} > 0 \leftrightarrow C'(1-s) > \frac{t}{4}$.

b) $\frac{\partial \pi^C}{\partial s} = \frac{C'(1-s)}{2} - \frac{t}{8} > 0 \leftrightarrow C'(1-s) > \frac{t}{4}$.

c) $\frac{\partial CS^C}{\partial s} = \frac{t}{8} > 0$.

d) $\frac{\partial W^C}{\partial s} = C'(1-s) - \frac{t}{8} > 0 \leftrightarrow C'(1-s) > \frac{t}{8}$.

3.3 Deviation Profits

A firm deviates from a collusion agreement if it is profitable to do so. In this case, it can set a lower price and capture a fraction of the market if the rival's price is low or the whole market if the rival's price is high. If a firm decides to capture the whole market, it sets a price that induces the consumer that most dislikes its product indifferent between both products. Therefore, the optimal deviation price is given by

$$p_i\left(p_j\right) = \begin{cases} \frac{p_j}{2} + \frac{st}{4} & \text{if} \quad p_j \leq \frac{3}{2}st \\ p_j - \frac{st}{2} & \text{if} \quad p_j \geq \frac{3}{2}st \end{cases}$$

Given the collusion prices $\left(p_1^C, p_2^C\right)$, the optimal deviation price and profit are:[3]

$$p^D = \begin{cases} \frac{v}{2} + \frac{st-4C(1-s)}{8} & \text{if} \quad v \leq C\left(1-s\right) + \frac{7}{4}st \\ v - \frac{4C(1-s)+3st}{4} & \text{if} \quad v \geq C\left(1-s\right) + \frac{7}{4}st \end{cases}$$

$$\pi^D = \begin{cases} \frac{(4v-4C(1-s)+st)^2}{64st} & \text{if} \quad v \leq C\left(1-s\right) + \frac{7}{4}st \\ v - \frac{4C(1-s)+3st}{4} & \text{if} \quad v \geq C\left(1-s\right) + \frac{7}{4}st \end{cases}$$

The effect of product design on deviation profit is ambiguous regardless of whether or not the deviating firm captures a fraction of the market.[4] In particular, it is negative if the marginal disutility from consuming a less customized product is low enough.

Proposition 4 *If a firm deviates from the collusive agreement, the following is found:*

a) when $v \leq C\left(1-s\right) + \frac{7}{4}st$, $\frac{\partial p^D}{\partial s} > 0$, *otherwise,* $\frac{\partial p^D}{\partial s} < 0 \longleftrightarrow c'\left(1-s\right) < \frac{3t}{4}$.

b) when $v \leq C\left(1-s\right) + \frac{7}{4}st$, $\frac{\partial \pi^D}{\partial s} < 0 \longleftrightarrow C'\left(1-s\right) < \frac{4v-4C(1-s)-st}{8s}$, *otherwise,* $\frac{\partial \pi^D}{\partial s} < 0 \longleftrightarrow C'\left(1-s\right) < \frac{3t}{4}$.

Proof: see Appendix.

3.4 Critical discount factor

As can be seen in Proposition 5, a firm decides to deviate from the collusive agreement when it undervalue future profits, i.e. when its discount factor is low enough.

Proposition 5 *Collusion is sustainable as an SPE if and only if*

$$\delta \geq \underline{\delta} = \frac{\pi^D - \pi^C}{\pi^D - \pi^N} = \begin{cases} \frac{4(v-C(1-s))-3st}{4(v-C(1-s))+5st} & \text{if} \quad v \leq C\left(1-s\right) + \frac{7}{4}st \\ \frac{4(v-C(1-s))-5st}{8((v-C(1-s))-st)} & \text{if} \quad v \geq C\left(1-s\right) + \frac{7}{4}st, \end{cases} \tag{7}$$

where $\underline{\delta}$ *represents the lowest discount factor that is needed to sustain collusion between firms.*[5]

[3] Throughout the paper we assume that $v \geq C\left(1-s\right) + \frac{3}{4}st$. Therefore, the case in which the deviating firm captures a fraction of the market is possible because $C\left(1-s\right) + \frac{7}{4}st \geq C\left(1-s\right) + \frac{3}{4}st$.

[4] This happens when $v \leq C\left(1-s\right) + \frac{7}{4}st$.

[5] This condition is obtained from inequality (4).

We now analyze the effects on the set of discount factor values over which collusion can arise, $[\underline{\delta}, 1]$. If $\underline{\delta}$ decreases, this set expands, so that collusion is easier. But, collusion is more difficult to sustain when $\underline{\delta}$ increases. From Proposition 6, the effect of product design depends on the marginal disutility of consuming a less generic product. In particular, we find that if this marginal disutility is low enough a more customized product eases collusion; otherwise it hinders collusion if the consumer's utility obtained from buying the fully targeted design of her/his preferred product is low enough. This result is mainly explained by the effect of product design on deviation profit. In particular, if the marginal disutility is low enough a more customized product reduces deviation profit, which increases the incentive to collude.

Proposition 6 *Under Assumption 1, the lowest discount factor that is needed to sustain collusion is decreasing on s if the marginal disutility of consuming a less generic product is low enough; otherwise, it is increasing if v is low enough.*

$$\frac{\partial \underline{\delta}}{\partial s} < 0 \ \textit{if}\, C'(1-s) < \frac{3}{4}t.$$

Proof: see Appendix.

As the product is more personalized and, consequently, more different from the rival product, collusion is easier to maintain as in the models à la Hotelling (1929) that analyze collusion (Chang (1991), Häckner (1996)). This happens when the marginal vertical cost is low enough. Otherwise, I find that collusion is easier to sustain the more similar the products are, as in vertical differentiation models (Häckner (1994)).

4 Conclusions

In this paper we analyze the effect of customizing a product on the ability of firms to tacitly collude on prices. Following Bar-Isaac et al. (2014), we allow firms to be located inside the circle in the Salop model (1979). Thus, a product is characterized by two dimensions: a horizontal dimension that reflects the variety of a product and a vertical dimension that reflects the degree of customization of a product.

Our analysis shows that the effect of customizing a product on the stability of collusion depends on the sensitivity of consumers' utility to the degree of customization, which is represented by the marginal disutility of consuming a less generic product. In particular, if this sensitivity is low enough greater customization facilitates collusion. Otherwise it hinders collusion if consumers value the product little.

From our welfare analysis we conclude that under collusion the effects of customizing depend on the sensitivity of consumers' utility to the degree of customization, but the effect on the consumer surplus is always positive. Thus, if this sensitivity is low enough a more customized product involves a lower price, profit and welfare. Otherwise, the opposite result is obtained. On the other hand, at the stage of punishment for deviating from the collusion price, which is modeled as a duopoly, it is obtained that a more customized product implies a higher price, which in turn leads to a higher profit. However, the effect of customizing on consumer surplus and welfare is negative if the sensitivity of consumers' utility to the degree of customization is low enough, but positive if it is high enough.

Appendix

Proof of Proposition 2. We make the conjecture that the market is fully covered, so firms set prices in such a way that the consumer who is indifferent between buying both products ($\widehat{x}$) obtains no utility if he/she buys any products. Thus, $p_1 = v - C(1-s) - ts\widehat{x}$ and $p_2 = v - C(1-s) - ts(1/2 - \widehat{x})$. The joint profit of the firms is:

$$\begin{aligned}\pi(\widehat{x}) &= p_1 D_1 + p_2 D_2 = p_1 2\widehat{x} + p_2(1-2\widehat{x}) \\ &= (v - C(1-s) - ts\widehat{x})\, 2\widehat{x} + (v - C(1-s) - ts(1/2-\widehat{x}))(1-2\widehat{x})\end{aligned}$$

From the first order condition we obtain that $\widehat{x} = 1/4$ maximizes the joint profit.

We show here that our conjecture of the market being fully covered is correct. If prices are higher than $\left(p_1^C, p_2^C\right)$, then the market is partially covered because those consumers located at around the angle 1/4 and 3/4 of circumference do not buy any products. Firms set prices in such a way that the consumers ($\widehat{x}_1$ and $\widehat{x}_2$) who are indifferent between buying a product and not buying any products obtain no utility if they buy the product. Thus $v - C(1-s) - ts\widehat{x}_1 - p_1 = 0$ and $v - C(1-s) - ts(1/2 - \widehat{x}_2) - p_2 = 0$, and the number of buyers of each product is:

$$\begin{aligned}D_1 &= 2\widehat{x}_1 = \frac{2(v - C(1-s) - p_1)}{ts} \text{ and} \\ D_2 &= 1 - 2\widehat{x}_2 = \frac{2(v - C(1-s) - p_2)}{st}\end{aligned}$$

When the market is partially covered the joint profit function and the first order conditions are:

$$\pi(.) = p_1 \frac{2(v - C(1-s) - p_1)}{ts} + p_2 \frac{2(v - C(1-s) - p_2)}{st}$$

$$\frac{\partial \pi(p_1, p_2)}{\partial p_i} = \frac{4(v - C(1-s) - 2p_i)}{2st}$$

Taking collusion prices into account the following is obtained:

$$\frac{\partial \pi\left(p_1^C, p_2^C\right)}{\partial p_i} = -\frac{4(v - C(1-s)) - 2st}{2st} < 0 \leftrightarrow v > C(1-s) + \frac{st}{2}$$

Under Assumption 1 we find that $\frac{\partial\pi\left(p_1^C,p_2^C\right)}{\partial p_i} < 0$ since $C(1-s)+\frac{3}{4}st > C(1-s)+\frac{st}{2}$.

Therefore, firms have no incentive to raise prices above $\left(p_1^C,p_2^C\right)$, and the market is fully covered. Given that $\widehat{x}=1/4$, the prices and profits are (6). ■

Proof of Proposition 4. We obtain:

$$\frac{\partial p^D}{\partial s}=\begin{cases}\frac{t+4C'(1-s)}{8}>0 & \text{if} \quad v\leq C(1-s)+\frac{7}{4}st\\ \frac{4C'(1-s)-3t}{4}<0\longleftrightarrow C'(1-s)<\frac{3t}{4} & \text{if} \quad v\geq C(1-s)+\frac{7}{4}st\end{cases}$$

$$\frac{\partial\pi^D}{\partial s}=\begin{cases}\begin{array}{c}-\frac{(4v-4C(1-s)+st)\left(4v-4C(1-s)-st-8C'(1-s)s\right)}{64ts^2}<0\\ \longleftrightarrow C'(1-s)<\frac{4v-4C(1-s)-st}{8s}\end{array} & \text{if} \quad v\leq C(1-s)+\frac{7}{4}st\\ \frac{4C'(1-s)-3t}{4}<0\longleftrightarrow C'(1-s)<\frac{3t}{4} & \text{if} \quad v\geq C(1-s)+\frac{7}{4}st\end{cases}$$

■

Proof of Proposition 6. We have that:

$$\frac{\partial\delta}{\partial s}=\begin{cases}\frac{-32t}{(4v-4C(1-s)+5st)^2}\left(v-C(1-s)-sC'(1-s)\right) & \text{if} \quad v\leq C(1-s)+\frac{7}{4}st\\ \frac{-8t}{64(v-C(1-s)-st)^2}\left(v-C(1-s)-sC'(1-s)\right) & \text{if} \quad v\geq C(1-s)+\frac{7}{4}st.\end{cases}$$

$\frac{\partial\delta}{\partial s}<0$ if and only if $v>sC'(1-s)+C(1-s)$. Under Assumption 1, the condition $v>sC'(1-s)+C(1-s)$ is satisfied if $C(1-s)+\frac{3}{4}st>sC'(1-s)+C(1-s)$, which is equivalent to $C'(1-s)<\frac{3}{4}t$. Otherwise, $\frac{\partial\delta}{\partial s}$ is positive if $v<sC'(1-s)+C(1-s)$. ■

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