Resource Effect in the Core-Periphery Model

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Abstract

This paper develops an extension of the Core-Periphery (CP) model (Krugman, 1991) by considering a competitive primary sector that extracts a renewable natural resource. The dynamics of the resource gives rise to a new dispersion force: the resource effect. If primary goods are not tradable, lower trade costs boost dispersion, and the agglomeration-dispersion transition is sudden or smooth depending on the productivity of the primary sector. Cyclic behaviors arise for high levels of productivity in resource extraction. If primary goods are tradable, in most cases, the symmetric equilibrium goes from stable to unstable as the openness of trade increases.

JEL classification: F12, F18, R12, Q01

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1 Introduction

The NEG literature has mainly focused on industrialized economies, overlooking rural or resource based economies. However, of the 80 million migrants worldwide in 1990, 25 million migrated for environmental reasons or because of resource degradation (Carr, 2009). Many of these migratory movements originated in rural or developing countries. Since the middle of the 20th century, about 1.2 billion hectares of land in the world have suffered soil degradation, with the consequent declines in yields and harvests, so causing massive numbers of environment-induced migrants (Swain, 1996). These migratory processes have important consequences on the spatial distribution of the economic activity, and an analysis of their provoking forces is merited. This is the aim of this paper, which extends the benchmark

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Core-Periphery model (CP model) (Krugman, 1991) by incorporating a (renewable) natural resource.

There are a number of well documented examples of migration and redistribution of economic activity motivated by the depletion of renewable natural resources. Kirby (2004) describes the geographic movements of fleets and main harbours in the exploitation of oyster fisheries along the coasts in eastern and western North America, and eastern Australia. Andrew et al. (2003) reports how, in Chile, the reduction in the biomass and overexploitation of the sea urchin led to the appearance of new fleets, ports and processing facilities in the south, while the harvesting of the resource tended to diminish in the middle regions of the country. After several years of rapid expansion of the fisheries into the southernmost region, due to the renewing ability of the resource, the proportional contribution to the national harvest of the middle regions began to recover, which boosted the economic activity in the region again. In Madagascar, farmers clear their land with 'slash and burn' strategies, which lead to deforestation and soil degradation. They proceed to cultivate the land for a couple of years until the soil is exhausted, after which they move on to new unexploited lands (Jouanjean et al., 2014). Other examples can be found for Brazil, the Dominican Republic, Nicaragua and Costa Rica (Carr, 2009; Chambron, 1999) and for Guatemala and Sudan (Bilsborrow and DeLargy, 1990). Anderson et al. (2011) provide an overview of the exploitation of the sea cucumber fisheries where the same behavioral pattern is observed: resource degradation in highly agglomerated regions triggers a process that forces population and economic activity away to new unexploited regions.

The resulting dispersion process depends heavily on the resource: its regenerative ability, the harvesting effort and the techniques used. These elements are not taken into account in NEG models (designed mainly for industrialized economies), where the only dispersion effects arise from the competition among industrial firms and the existence of transport costs. A comprehensive analysis should also take into account the effects of environment and resource degradation.

Transport cost is an important element in the NEG literature and it also plays an important role in the development of rural economies. Reduction in transportation costs, construction of new roads and infrastructures all facilitate access to distant regions. The profitability of the exploitation of natural resources in far away areas increases, which allows the expansion of the economic activity. For example, a curious land-use dynamics took place in Laos, the Philippines and Amazonia, where landowners intensified agriculture activities close to new or improved roads. At the same time, forests began to regenerate in regions farther away from the roads (Laurance et al., 2009). Reymondin et al. (2013) studies five infrastructure projects for Brazil, Paraguay, Peru, Panama and Bolivia, where these new roads led to forest exploitation, deforestation and expansion of the agricultural frontier to new, unexploited regions. Furthermore, in Brazil, Pfaff (1999) and in Bolivia, Kaimowitz, et al. (2002), highlight that unexploited soil of better quality together with new roads increased the probability of deforestation in order to expand agricultural exploitations for Brazil and Bolivia, respectively.

Therefore, the resulting spatial structure of the economic activity depends on the interaction between transport costs and the resource dynamics. Lower transport costs facilitate trade, which increases the profitability of exploiting distant areas, so encouraging migration and spatial expansion of the economic activity. Additionally, areas whose exploitation has declined, due to the shift in the economic activity, tend to experience a regeneration of their natural resources. Thus, a reduction in transport costs reinforces the dispersion effect driven by the resource dynamics.

Helpman (1998) studies how a fixed endowment (land) boosts the dispersion of the economic activity. Some extensions of this model are found in Suedekum (2006), Pflüger and Südekum (2008), Pflüger and Tabuchi (2010), Leite et al. (2013) and Cerina and Mureddu (2014). These models adjust well for industrialized economies, where congestion and competition for land (a fixed resource) is the driving force of dispersion. Population is the only dynamic factor. However, it does not seem sufficient for regions that base their economic activity on dynamic/renewable natural resources. In resource based economies the dispersion depends on two fundamental aspects: how the exploitation of the natural resource takes place and how well this resource regenerates itself. Thus, population and resource dynamics interact. An agglomerated equilibrium may be stable if the resource endowment is fixed, while it becomes unstable once the dynamics of the resource is taken into account. The resulting spatial distribution of the economic activity is completely different.

There have been some attempts to incorporate notions from environmental economics into NEG models. Pflüger (2001) studies the option of imposing taxes on emissions; Zeng and Zhao (2009) and Rauscher (2009) extend some NEG models to study the impact of pollution on the spatial configuration of the economy; Rieber and Tran (2009) investigate the consequences of unilateral environmental regulations; and Rauscher an Barbier (2010) highlight the conflict arising from competition for space between economic and ecological systems. Other attempts to shift the focus from the industrial sector to other sectors of an economy are Lanaspa and Sanz (1999), Berliant and Kung (2009), and Sidorov and Zhelobodko (2013). However, the regenerative ability of natural resources and the extractive efficiency of harvesting efforts is not considered.

To the best of our knowledge the literature has not incorporated these elements in NEG models. We modify the original CP model by introducing the dynamics of a renewable natural resource, which is extracted as a primary good (Clark, 1990; Vardas and Xepapadeas, 2015), and the double function of primary goods, both as an input for industrial production and as a final consumption good (Pflüger and Tabuchi, 2010). We assume that agents are myopic, that is, they extract the resource without taking into account its dynamics. This set-up is the most consistent with the examples found in the literature.

In our model, industrial goods are produced using the primary good as raw material and there is free labor mobility between sectors and regions. We study both non-tradable primary goods (fertile land, drinking water or perishable natural goods) and tradable primary goods (agricultural goods). Under the assumptions of non-tradability of the primary good and free labor mobility across sectors, the market size effect dominates the competition effect, as in Helpman (1998). Then, the renewable natural resource and its dynamics are the main mechanisms that drive dispersion, giving rise to the resource effect.The effect of transport cost on the stability of dispersion and agglomeration is reverted. This is compatible with the pattern described by the empirical literature: lower transport costs and resource degradation encourage migrations process and the distribution of the economic activity.

The extraction productivity of the primary sector determines the strength of the resource effect, determining how the transition from agglomeration to dispersion takes place. When dispersion forces are weak (low extraction productivity) there is an abrupt transition from agglomeration to dispersion, as the cases of the fishery industry pointed out before. When dispersion forces are strong, a smooth transition can take place, like the reported cases of slow depopulation driven by de decline in soil fertility. Moreover, if dispersion forces are strong enough (relative to transport costs), cyclical behavior may arise: an agglomeration process raises the primary demand, so encouraging larger extractions that compromise the long-term level of the resource and its future extractions. Later, this primary price increases sufficiently to revert the migration process. This is compatible with the chase-and-flee cycle of Rauscher (2009) in the environmental literature, and also with the definitions of circular migration in the migration and economic labor literature (Newland, 2009).

If the primary good is tradable, the openness of trade affects the traditional dispersion-agglomeration forces and also the strength of the new one linked to the resource and its dynamics. Numerical analysis highlights some regularities. First, as the primary good becomes more tradable, the advantage of being in the region with the higher sustainable level of resource is reduced, which weakens the associated dispersion forces. Second, the predominant pattern observed for the symmetric equilibrium is the one that goes from stable to unstable as transport costs decreases.

The paper is organized as follows: Section 2 introduces the model, Section 3 studies the case of non-tradable primary goods and in Section 4 trade of primary goods is allowed. Section 5 concludes.

2 The Model

A world with two regions ($j = 1, 2$) is considered. Two kinds of goods are assumed: manufactures, produced by an increasing-returns sector that can be located in either region, and a primary good that is extracted or harvested from a resource endowment by competitive firms in each region. The industrial sector uses two inputs to produce manufactures: labor and primary good. The primary sector uses only labor for the extraction of the resource. Hereinafter, the extracted goods from the primary sector will be called primary goods when their destination is to be consumed, and raw materials when their destination is to be used as inputs. Finally, to incorporate the dynamics of the natural resource and its relation with economic activity in a simple way, we assume that there is free labor mobility between industrial and primary sector. This assumption makes the model extremely tractable. Moreover, if there were no mobility at all between sectors, the dynamics of the resource and its long-run stock, as will be shown later, would be independent of changes in economic activity. Then, the model would be similar to Helpman (1998) and Pflüger and Tabuchi (2010).[1]

2.1 Households

Households seek to maximize their utility, which takes the form of a nested Cobb-Douglas (across sectors) and CES (over the varieties) used in the original Krugman model (1991). Thus, a representative household in region 1 solves the following consumption problem:

$$\max_{c_{1i},c_{2i},c_{H_1},c_{H_2}} U_1 = \ln\left[C_{M_1}^{\mu} C_{H_1}^{1-\mu}\right] \tag{1}$$

$$\text{s.t. } w_1 = \int_0^{n_1} c_{1i}p_{1i}di + \int_0^{n_2} c_{2i}p_{2i}\tau di + p_{H_1}c_{H_1} + p_{H_2}c_{H_2}\nu \tag{2}$$

with parameter $\mu \in (0,1)$ and

$$C_{M_1} = \left(\int_0^{n_1} c_{1i}^{\frac{\sigma-1}{\sigma}} di + \int_0^{n_2} c_{2i}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

$$C_{H_1} = \left(c_{H_1}^{\frac{\sigma-1}{\sigma}} + c_{H_2}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

where C_{M_1} and C_{H_1} are consumption indexes of industrial and primary goods respectively with $\sigma > 1$ (for simplicity we assume the same elasticity of substitution for both sectors); c_{ji} is the consumption of variety i produced in region j ($j = 1, 2$); n_j is the number of varieties existing in region j; because of free labor mobility, the salary is the same in both sectors and w_j is the income per household in region j; c_{H_1} and c_{H_2} are the consumptions of the primary or harvested good extracted in regions 1 and 2 respectively (Fujita et al., 2001, ch. 7); p_{ji} is the (fob) price of the variety i of the industrial good produced in region j; $\tau > 1$ and $\nu > 1$ are iceberg transport costs of industrial and primary goods, respectively; and finally, p_{H_j} is the price of the primary good of region j. The mirror-image problem is solved for households in region 2.

[1] The mobility of labor between sectors has been addressed by Puga (1998). The author assumes free mobility across sectors and regions. Labor dynamics in a specific sector of a region is driven by the relation between the real wage in that sector and a weighted average of the real wages in the other sector (within the same region) and real wages in the other region. In our paper, for the sake of simplicity, we have assumed that nominal wages within a region adjust immediately to become equal in the two sectors. Although the dynamics would be more complex, the steady states equilibria would remain the same.

From the first order conditions of the maximization problem (1)-(2), the following demand functions are obtained:

$$c_{1i} = C_{M_1}\left(\frac{p_{1i}}{P_1}\right)^{-\sigma}, c_{2i} = C_{M_1}\left(\frac{p_{2i}\tau}{P_1}\right)^{-\sigma} \text{ with } C_{M_1} = \frac{\mu w_1}{P_1} \quad (5)$$

$$c_{H_1} = C_{H_1}\left(\frac{p_{H_1}}{P_{H_1}}\right)^{-\sigma}, c_{H_2} = C_{H_1}\left(\frac{p_{H_2}\nu}{P_{H_1}}\right)^{-\sigma} \text{ with } C_{H_1} = \frac{(1-\mu)\, w_1}{P_{H_1}} \quad (6)$$

where P_1 and P_{H_1} are the industrial and primary price indexes for region 1, that is,

$$P_1 = \left(\int_0^{n_1} p_{1i}^{1-\sigma} di + \int_0^{n_2} (p_{2i}\tau)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \quad (7)$$

$$P_{H_1} = \left[p_{H_1}^{1-\sigma} + (p_{H_2}\nu)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \quad (8)$$

Mirror-image formulas for P_2 and P_{H_2} hold for consumers in region 2.

2.2 Primary Sector

In the natural extractive sector, a primary firm seeks to maximize its benefits, in a perfect competitive market, choosing the amount of labor to employ in the extraction of the resource, subject to the extraction function for region j, given by

$$H_j = \epsilon S_j L_{H_j}, \quad \epsilon > 0 \quad (9)$$

where S_j is the available stock of the natural resource, L_{H_j} is the labor employed in the primary sector and ϵ is a productivity parameter in the extraction, assumed to be equal for both regions for the sake of simplicity. As is usual in environmental economic models, the productivity of labor depends positively on the available stock of the natural resource S_j. Firms are myopic, that is, they extract the resource without taking into account its dynamics. The extracted or harvested resource, H_j, can be consumed or used as a raw material for industrial production. The maximization of profits, in a competitive market with free entry, needs the following condition,[2]

$$p_{H_j} = w_j \frac{L_{H_j}}{H_j} = \frac{w_j}{\epsilon S_j} \quad (10)$$

[2] As reported by Adhikari et al. (2004) there still exist some examples of free access to forest resources in Nepal. Moreover, fisheries have proven difficult to regulate and an open-access externality of reasonable size still exists in Nordic Fisheries (Waldo et al., 2016). Poor regulation has resulted in both stock depletion and low economic returns, leading to the well known "Tragedy of the commons". Fishery, forestry, irrigation, water management, animal husbandry, biodiversity and climate change are the usual areas where the "Tragedy" has arisen (Laerhoven, 2007).

where p_{H_j} is the price of the primary good and w_j is the salary in region j.

2.3 Industrial Sector

A firm in the industrial sector employs labor and raw materials to produce industrial goods, according to the production function

$$x_{ji} = \left(\frac{1}{\beta}\right)\left(l_{x_{ji}} - f\right)^{\alpha} h_{ji}^{1-\alpha}, \quad 0 < \alpha < 1 \tag{11}$$

$$h_{ji} = \left(h_{1ji}^{\frac{\sigma-1}{\sigma}} + h_{2ji}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{12}$$

where $l_{x_{ji}}$ is labor used in producing variety i in region j, and x_{ji} is the output; h_{ji} is an index of raw materials employed in the production of variety i in region j; h_{kji} is the primary good extracted in region k employed in region j production of variety i. For the sake of simplicity we have assumed same elasticity substitution σ for primary goods. Parameter $\beta > 0$ is the marginal input requirement and f is a fixed cost. Note that if $\alpha = 1$, the production function (11) is the same as the one proposed by Krugman (1980, 1991), which involves a constant marginal cost and a fixed cost, giving rise to economies of scale. When $\alpha \in (0,1)$ the use of the raw material is necessary for production and increases labor productivity.

It is assumed that there are a large number of manufacturing firms, each producing a single product in monopolistic competition (Dixit and Stiglitz, 1977). Given the definition of the manufacturing aggregate (3), the elasticity of demand facing any individual firm is $-\sigma$. Then, the profit-maximizing price behavior of a representative firm in region j is

$$p_{ji} = \frac{\sigma}{\sigma - 1}\beta\left(\frac{w_j}{\alpha}\right)^{\alpha}\left(\frac{P_{H_j}}{1-\alpha}\right)^{1-\alpha} \tag{13}$$

Since firms are identical and face the same wage and the same price of raw materials within a region, manufactured good prices are equal for all varieties in each region, so the subscript i can be dropped. Consequently, p_j $(j = 1, 2)$ will refer to region j specific industrial good price. Equally, resource demand is equal for all firms in the same region j, so we shall name the region j specific resource and labor demands per firm h_j and l_{x_j} $(j = 1, 2)$. Primary goods demand functions for the industrial sector in region 1 are

$$h_{11} = h_1^{1-\sigma}\left(\frac{1-\alpha}{\alpha}\frac{w_1\left(l_{x_1} - f\right)}{p_{H_1}}\right)^{\sigma} \quad \text{and} \quad h_{21} = h_1^{1-\sigma}\left(\frac{1-\alpha}{\alpha}\frac{w_1\left(l_{x_1} - f\right)}{\nu p_{H_2}}\right)^{\sigma} \tag{14}$$

and for region 2, h_{12} and h_{22} are mirror images of (14). Therefore,

$$h_j = \frac{1-\alpha}{\alpha} \frac{w_j}{P_{H_j}} \left(l_{x_j} - f \right) \text{ for } j = 1, 2. \tag{15}$$

Comparing the prices of representative products in (13), we have

$$p \equiv \frac{p_1}{p_2} = \left(\frac{w_1}{w_2} \right)^{\alpha} \left(\frac{P_{H_1}}{P_{H_2}} \right)^{1-\alpha} \tag{16}$$

Because there is free entry in the industrial sector, a firm's profits must equal zero. Using this condition and (13) and (15), it is obtained that

$$x_j = f \frac{\sigma - 1}{\beta} \alpha^{\alpha} (1-\alpha)^{1-\alpha} \left(\frac{w_j}{P_{H_j}} \right)^{1-\alpha} \tag{17}$$

$$l_{x_j} = f \left[1 + \alpha (\sigma - 1) \right]. \tag{18}$$

The aggregate labor employed in the industrial sector of region j is $L_{E_j} = n_j f \left[1 + \alpha (\sigma - 1) \right]$. Here again, if $\alpha = 1$, we obtain the same expression as in Krugman's model.

2.4 Dynamics

Natural Resources: The regions are assumed to be endowed with a renewable natural resource (S_j) whose dynamics follows a logistic growth function (Clark, 1990)

$$\dot{S}_j = g S_j \left(1 - \frac{S_j}{CC} \right) - H_j \tag{19}$$

where $g > 0$ is the intrinsic growth rate of the resource, that is, the rate at which the natural resource regenerates itself. The carrying capacity, $CC > 0$, is the maximum size of the resource that can be sustained. Because we are studying symmetric regions, both g and CC are assumed to be equal in both regions, which simplifies the model. Taking into account H_j, given by (9), into (19), the sustainable level of the resource (the positive steady-state level) is given by

$$S_j^* = \left(1 - \frac{\epsilon}{g} L_{H_j} \right) CC > 0 \text{ if and only if } L_{H_j} < g/\epsilon \tag{20}$$

which is globally stable for a given value of L_{H_j}. Otherwise, the only globally stable steady state is the null one.

Population Mobility: Workers are mobile between regions and choose to migrate if they gain in terms of individual welfare from doing so. We assume that $L_1 + L_2 = 1$ and, as is usual in NEG models, population reallocation follows the following dynamics:

$$\dot{L}_1 = L_1 (1 - L_1) \left(\frac{V_1}{V_2} - 1 \right) \tag{21}$$

where V_j is the indirect utility, defined as the ratio of nominal wage w_j to the Cobb-Douglas average price index across sectors (Sidorov and Zhelobodko, 2013; Forslid and Ottaviano, 2003),[3]

$$V_j = \frac{w_j}{(P_j)^{\mu} \left(P_{H_j}\right)^{1-\mu}} \tag{22}$$

Therefore, the dynamic of the model will be driven by the differential system (19) for $j = 1, 2$ and (21).

3 A non-tradable primary good

In this section we present the case of a non-tradable primary good. This is the case of fertile land, drinking water or highly perishable products, for example. To do this we assume that $\nu \to \infty$. When primary trade costs are unaffordable, households and firms can only purchase primary goods extracted in the local region. Thus, index prices P_{H_1} and P_{H_2}, defined in (8) and in its mirror image formula for region 2, and (16), become

$$P_{H_j} = p_{H_j} \text{ for } j = 1, 2 \tag{23}$$

$$p = \left(\frac{w_1}{w_2} \right) \left(\frac{S_1}{S_2} \right)^{-(1-\alpha)} \tag{24}$$

where (10) has been taken into account. From (6) we have that, in region 1, household´s demand of primary good is

$$c_{H_2} = 0 \text{ and } c_{H_1} = C_{H_1} = \frac{(1-\mu)\, w_1}{p_{H_1}} \tag{25}$$

Mirror-image formulas hold for consumers in region 2.

Demand equations (14)-(15) can be simplified:

$$h_{12} = h_{21} = 0 \text{ and } h_j = \frac{1-\alpha}{\alpha} \frac{w_j}{p_{H_j}} \left(l_{x_j} - f\right) \text{ for } j = 1, 2. \tag{26}$$

[3] In Fujita et al. (2001) $\dot{L}_1 = L_1(V_1 - \bar{V})$ where $\bar{V} = L_1 V_1 + (1 - L_1) V_2$ is the weighted average of indirect utilities. A simple manipulation derives that $\dot{L}_1 = L_1(1 - L_1)(V_1 - V_2)$, which is equivalent to (21) if we divide by V_2.

3.1 Short-Run Equilibrium

In the short-run equilibrium, households maximize their utility, industrial and primary firms maximize their profits, there is free entry in both sectors, and market clearing conditions hold for the three markets: labor, primary and industrial goods.

As a result of the free labor mobility assumption, the labor market clearing condition states that

$$L_j = \int_0^{n_j} l_{x_{ij}} di + L_{H_j} = L_{E_j} + L_{H_j} \tag{27}$$

where L_j is the total population of region j.

In the primary sector, total harvesting, H_j, must satisfy the demand for final consumption of the primary good (25) and the demands of the industrial firms for raw materials (26), that is,

$$H_j = L_j \frac{(1-\mu)\, w_j}{p_{H_j}} + n_j \frac{1-\alpha}{\alpha} \frac{w_j}{p_{H_j}} \left(l_{x_j} - f \right) \tag{28}$$

Using equations (10), (18), and (27) we have that the primary sector clearing condition (28) implies

$$L_{H_j} = \frac{\sigma - \mu \left[1 + \alpha \left(\sigma - 1\right)\right]}{\sigma} L_j \tag{29}$$

$$L_{E_j} = \frac{\mu \left[1 + \alpha \left(\sigma - 1\right)\right]}{\sigma} L_j \tag{30}$$

$$n_j = \frac{\mu}{\sigma f} L_j \tag{31}$$

Trade is balanced if and only if the following equation is satisfied[4]:

$$TB = p \left(\frac{S_1}{S_2} \right)^{1-\alpha} \left(1 + \frac{L_1}{1 - L_1} p^{1-\sigma} \phi \right) - p^{1-\sigma} \left(\phi + \frac{L_1}{1 - L_1} p^{1-\sigma} \right) = 0 \tag{32}$$

where $\phi \equiv \tau^{1-\sigma}$, with $\phi \in (0, 1)$ is an index of the openness of trade. This equation has a unique positive solution. Using this solution and equation (24), the ratio of nominal wages can be obtained as a function of ϕ, L_1, S_1 and S_2.

As we move from the short-run to long-run equilibrium, however, some other features need to be taken into account. Workers are not interested in nominal wages but in real wages and they will migrate to the region with the highest welfare. Additionally, an increase in

[4] This equation is obtained in the online Appendix A, available for readers interested in these details at the website of the journal. From now on, all the long derivations and proofs are presented in online appendixes.

population will boost the use of natural resources for consumption and production, which provokes a dynamic adjustment of the natural environment. These two dynamic processes are explained in the following section.

3.2 Long-Run Equilibria

The usual agglomeration and dispersion forces of NEG literature (market size effect, Competition and Price index effects) arise in the model. As in Helpman (1998), a consequence of the nontradability of the primary good and the free labor mobility is that the market size effect always dominates the competition effect. In addition, as a result of the dynamics of the natural resources, a new dispersion force arises, as is proved in Proposition 1.

Note that, for a given level of L_j, the globally stable steady state value of the natural resource, given in (20), becomes

$$S_j^* = (1 - \epsilon\theta L_j)\, CC \text{ if } \epsilon\theta L_j < 1 \text{ otherwise } S_j^* = 0 \tag{33}$$

where

$$\theta \equiv \frac{\sigma - \mu\,[1 + \alpha\,(\sigma - 1)]}{g\sigma} \tag{34}$$

and (29) have been used.

Thus, due to the role played by the workforce in the resource extraction, a higher population, L_j, tends to reduce the level of the sustainable natural resource ($S_j^* > 0$). The same can be said for the extractive productivity parameter in the primary sector, ϵ. The following proposition establishes the consequences for wages and primary good price.

Proposition 1 *When population increases in one region, natural resource dynamics leads to*

(i) *lower nominal wages and*

(ii) *increase the price of primary goods and the industrial price index in that region.*

Proof. See the online Appendix A. ■

This is the Resource Effect, and it has two channels that encourage dispersion through the consumers utility. Property (*i*) stands for the linkage between the primary and the industrial sector, and depends on α. Property (ii) affects the cost of living and its strength depends on μ.

Despite the similarities, the resource effect is different to the effects derived by Helpman (1998) and Ottaviano and Puga (1998). In these other models the dispersion forces are driven by region-specific supplies of the nontradable good (or factors), which are fixed stocks. Thus, an increase in the population diminishes the stock per capita (per firm), so raising the price. In the model developed in this paper, the primary good is extracted or produced; it is not fixed. An increase in the population does not change the ratio H_j/L_j, in the short-run, due to the simultaneous increase in the extractive labor force (see equations (9) and (29)). However, in the long-run, the steady states stock decreases, and so, therefore, does the ratio H_j/L_j. The resource dynamic is essential for the existence of the resource effect.

Equation (21) can be simplified by replacing (22), and making use of P_1 definition in (7), its mirror image for P_2, (10), (23), (24), (31) and (32). Thus, the dynamic evolutions of the stocks of the natural resource and population between the two regions are driven by equations (19) with $j = 1, 2$ and (21) and can be rewritten as

$$\dot{L}_1 = L_1(1 - L_1)\left[\left(\frac{S_1}{S_2}\right)^{1-\mu\alpha-\mu(1-\alpha)/(1-\sigma)} p^{\mu(1-2\sigma)/(1-\sigma)} - 1\right] \tag{35}$$

$$\dot{S}_1 = gS_1\left(1 - \frac{S_1}{CC}\right) - \epsilon g\theta S_1 L_1 \tag{36}$$

$$\dot{S}_2 = gS_2\left(1 - \frac{S_2}{CC}\right) - \epsilon g\theta S_2(1 - L_1) \tag{37}$$

where θ is defined by (34), and p is a function of the population size, according to equation (32).[5] A long-run equilibrium is a stationary point of the dynamic equation system (35)-(37), where workers do not have incentives to move from one region to the other and natural resource stocks remain constant. Furthermore, because we are studying a renewable natural resource, we are interested in the set of parameters that allows at least one long-run sustainable solution ($S_j^* > 0$). To ensure this, we assume hereinafter that

$$\epsilon\theta < 2. \tag{38}$$

[5] Note that after some manipulations we have that

$$\frac{P_1}{P_2} = \left(\frac{\frac{L_1}{L_2}p^{1-\sigma} + \phi}{\frac{L_1}{L_2}\phi p^{1-\sigma} + 1}\right)^{1/(1-\sigma)} = p^{\sigma/(1-\sigma)}\left(\frac{S_1}{S_2}\right)^{(1-\alpha)/(1-\sigma)}$$

If parameters satisfy the sustainability condition (38), then there exists a symmetric interior equilibrium, characterized by the following values, where population is equally distributed:

$$L_1^* = 1/2,\ S_1^* = S_2^* = \left(1 - \epsilon\frac{\theta}{2}\right) CC,\ p^* = 1 \tag{39}$$

Note that if $\epsilon\theta < 1$, the symmetric (interior) equilibrium coexists with the following two agglomeration (boundary) equilibria:

$$L_1^* = 1,\ \ S_1^* = (1-\epsilon\theta)\, CC,\ \ S_2^* = CC,\ \ p^* = \phi^{\frac{-1}{\sigma}} (1-\epsilon\theta)^{-(1-\alpha)/\sigma} \tag{40}$$

and

$$L_1^* = 0,\ \ S_1^* = CC,\ \ S_2^* = (1-\epsilon\theta)\, CC,\ \ p^* = \phi^{\frac{1}{\sigma}} (1-\epsilon\theta)^{(1-\alpha)/\sigma} \tag{41}$$

When $\epsilon\theta \geq 1$, the agglomeration equilibria become.[6]

$$L_1^* = 0,\ \ S_1^* = CC,\ \ S_2^* = 0,\ \ p^* = 0 \tag{42}$$

Mirror-image values are obtained for $L_2^* = 0$.

3.3 Stability Properties

According to (39)-(42), economic activity can be equally distributed between the regions or concentrated in one of them. Which equilibrium will prevail depends on the stability properties, expressed in terms of the parameters of the model, in the following proposition.

Proposition 2 *The symmetric interior equilibrium is locally stable if the following condition is satisfied:*

$$\phi > \max\{\phi^B, \phi^H\} \tag{43}$$

with $\phi^B \equiv 1 - \frac{(\sigma-1)(\sigma(1-\mu\alpha)-\mu(1-\alpha))\,\epsilon\theta}{(2\sigma-1)\mu(1-\epsilon\frac{\theta}{2})+(\sigma-1)(1-\mu-\mu(1-\alpha))\epsilon\frac{\theta}{2}}$ *and* $\phi^H \equiv 1 - \frac{2\sigma(\sigma-1)g\left(1-\epsilon\frac{\theta}{2}\right)}{(2\sigma-1)\mu+(\sigma-1)g(1-\epsilon\frac{\theta}{2})}$.

Meanwhile, the agglomeration equilibria (40)-(41) are locally stable (stable nodes) if the following condition holds:

$$\phi < \phi^S \equiv (1-\epsilon\theta)^{\frac{(\sigma-1)}{\mu(2\sigma-1)}[\sigma(1-\mu\alpha)-\mu(1-\alpha)]} \tag{44}$$

and agglomeration equilibrium (42) is always unstable.

[6] On replacing the symmetric equilibrium ($L_1^* = 1/2$, $S_1^* = S_2^* = (1-\epsilon\theta/2)CC$), and the agglomeration equilibria ($L_1^* = 1$, $S_1^* = (1-\epsilon\theta)CC$, $S_2^* = CC$ and $L_1^* = 0$, $S_1^* = CC$ and $S_2^* = (1-\epsilon\theta)CC$), in equation (32), the equilibrium price p^* is obtained. Thus, it is easy to confirm that the three differential equation system (35)-(37) vanishes for (39) and for (40)-(42).

Proof. See the online Appendix A. ■

In the previous proposition the superscript B is for "break" and the superscript S is for "sustain" (maintaining the name used by Fujita et al., 2001). The superscript H is for Hopf, since at this point a Hopf bifurcation arises, as will be shown later.

Condition (43) defines a region of stability for the symmetric equilibrium (shaded region in Figure 1) in the space of parameters ϵ and ϕ. This stability region is not empty, although it can be greater or smaller depending on the parameters of the model. The downward sloping curve ϕ^B in the (ϵ, ϕ) space, is the boundary for the pitchfork bifurcation, and ϕ^H, upward sloping, is the boundary for the Hopf bifurcation. Both curves intersect at point $\bar{\epsilon}$.[7]

Figure 1 represents the stability condition (43) for parameters $\sigma = 2$, $\alpha = 0.6$, $\mu = 0.8$, and $g = CC = 1$.

[Figure 1 - Stability region of the symmetric equilibrium]

The symmetric equilibrium (39) is not necessarily the only interior equilibrium for the system (35)-(37). According to the value of the parameters, there could be two more interior equilibria around the symmetric one. The following proposition proves this result.

Proposition 3 *If $\phi^B, \phi^S \in (0,1)$ and $\epsilon < \bar{\epsilon}$, an increase in the openness of trade leads to:*

i) *a sudden change from agglomeration to dispersion for low levels of the extractive productivity, $\epsilon < \min\{\tilde{\epsilon}, \bar{\epsilon}\}$.*

ii) *a smooth change from agglomeration to dispersion for high levels of the extractive productivity, $\tilde{\epsilon} < \epsilon < \bar{\epsilon}$.*

where $\tilde{\epsilon} > 0$ is the intersection point of ϕ^B and ϕ^S.

[7]
$$\bar{\epsilon} \equiv \frac{(\sigma-1)(1-\alpha) + \sigma(2+\theta) - \sqrt[2]{(\sigma-1)^2(1-\alpha)^2 + \theta\left[2(1-\alpha)(\sigma-1) + \sigma(2+\theta)\right]}}{\theta(\sigma + (\sigma-1)(1-\alpha))} > 0 \qquad (45)$$

Proof. See the online Appendix A. ■

Proposition 3 proves that as transport cost decreases (ϕ increases) the stability of the symmetric equilibrium changes. This prominence of transport costs is not new in NEG models. What is new in our model is the emergence of a second actor: the extraction productivity in the primary sector, measured by ϵ. This parameter will determine if the transition is sudden (a subcritical pitchfork bifurcation) or smooth (a supercritical pitchfork bifurcation). Both phenomena are observed in the real world. As reported by Andrew et al. (2003), the rapid movement of fishing efforts (fleet, fishermen, processing facilities, etc.) to unexploited regions has occurred in many world fisheries. In contrast, a slow depopulation has been observed as the consequence of deforestation or soil fertility decline (Dazzi and Lo papa, 2013). Parameter ϵ plays an important role in environmental economic models that use the catch-per-unit-effort resource production function. Its value depends on both the natural resource in question and the technology employed. Therefore, these two facts matter for a sudden or a smooth structural change in the geographical distribution of the economic activity.

Additionally, the incorporation of the dynamics of natural resources into the original core periphery model leads to the appearance of periodic solutions, as is proved in the following proposition.

Proposition 4 *When $\phi^i < \phi < \phi^H$ migration flows adopt a cyclic behavior,*

where

$$\phi^i \equiv 1 - 2\sigma \frac{\delta^i}{1 + \delta^i} \tag{46}$$

and δ^i is defined in the online Appendix A.

Proof. See Appendix A. ■

Note that ϕ^H and ϕ^i intersect at point $\bar{\epsilon}$. Thus, for $\epsilon > \bar{\epsilon}$ passing from the left to the right of curve ϕ^H, there are two complex conjugate eigenvalues that move from having negative real part to having positive real part, and the symmetric equilibrium loses its local stability. The proposition shows the emergence of cyclic behavior (a Hopf bifurcation) for relatively high values of the extractive productivity ($\epsilon > \bar{\epsilon}$).

The existence of cyclic behavior is new to the literature of CP models in continuous-time. However, the Policy Institute and the 2011 report of the European Migration Network recognize the existence of circular migration. In some cases it is due to environmental issues (Rauscher, 2009). Proposition 4 points again to the extraction productivity, ϵ, as a key parameter ($\epsilon > \bar{\epsilon}$), together with transport costs.

The following subsection describes the process of agglomeration-dispersion of the economic activity between two regions, focusing on the role of transport cost and primary sector productivity in the stability properties of the equilibria.

3.4 The role of transport cost and extraction productivity

The first result that stands out is that as the transport cost decreases (ϕ increases) the symmetric equilibrium changes from unstable to stable (Proposition 2). This is in contrast to the results found in the original CP model but in line with Krugman and Elizondo (1996), Helpman (1998) and Murata and Thisse (2005).

In the transition between instability and stability, the extraction productivity of the primary sector becomes important. As pointed out before, different values of ϵ can change the bifurcation pattern. Figure 1 gives a clear view of the role of ϵ. For a given transport cost, the larger the value of ϵ, the closer we are to the stability region of the symmetric equilibrium. So the dispersion force is a direct function of ϵ.

Note that this result is the opposite to what Tabuchi et al. (2016) find when they analyze an increase in the industrial productivity through a fall in the marginal labor requirement. Two differences are driving these opposite results. First, the model proposed by Tabuchi et al. (2016) has migration costs. This implies that the size of the gap between real wages matters in their model and not in ours. Thus, when industrial productivity increases in Tabuchi's model, the real wage gap widens, overcoming the effect of migration costs and giving place to further agglomeration. In our model the equilibrium implies real wage equalization, so an increase in the extraction productivity does not have a direct impact on prices through this channel.[8] Second, in our model, the extraction productivity has a

[8]Note that if in our model there were a real wage gap differential, and holding constant the stock of natural resources, an increase in ϵ would also widen this gap as in Tabuchi et al. (2016), and through the same direct channel. Nevertheless, the only possible equilibria where real wages are not equalized (in our

second channel through which it can affect the equilibria: it has an indirect impact through the long-run stock of natural resources. This channel is not present in Tabuchi et al. (2016). Thus, in the case of $L_1 \neq 1/2$ an increase in the extraction productivity can change the ratio of indirect utilities, shifting dynamics and the possible equilibria. Hence, while an increase in the industrial productivity tends to magnify regional disparities, an increase in the extraction productivity tends to mitigate or even revert these disparities when there are no migration costs.

The process depicted by Figure 2, the subcritical case, is characterized by a sudden change in spatial configuration (Fujita et al., 2001). This is because the non-symmetric interior equilibria connecting the agglomeration and symmetric solutions are unstable. In this case, the bifurcation diagram has a Krugman tomahawk shape, but the stability pattern is inverted.

[Figure 2 - Subcritical Pitchfork Bifurcation - parameter values: $\sigma = 2$, $\alpha = 0.6$, $\mu = 0.8$, $g = CC = 1$, and $\epsilon = 2.2$]

If the extraction productivity of the primary sector is low enough, a value of $\epsilon < \min\{\tilde{\epsilon}, \bar{\epsilon}\}$ (subcritical bifurcation), then dispersion forces are weak, and for low values of ϕ (such that $\phi^H < \phi < \phi^B$) agglomeration equilibria are stable. As transport costs decrease the equilibrium moves to the right in Figure 2, and a subcritical bifurcation takes place at ϕ^B. The peculiarity of this pattern is that for $\phi \in \left(\phi^B, \phi^S\right)$ both agglomeration and dispersion equilibria are locally stable. This occurs precisely because dispersion forces are weak, so when the distribution of the economic activity is near to being fully agglomerated, the size of the market can still overcome the dispersion forces, even at relatively low transport costs. However, when the distribution of the economic activity is near the symmetric equilibrium, the market size effect is not very strong because the difference between the sizes of the markets is small. So, dispersion forces can overcome agglomeration forces.

The process depicted by Figure 3, the supercritical case, is characterized by a smooth change in the spatial configuration. This is because the interior non-symmetric equilibria

model) are the agglomeration ones, where concentration has already reached its maximum.

are stable and connect the agglomeration and dispersion solutions. The bifurcation diagram closely resembles the one derived by Helpman (1998).

[Figure 3 - Supercritical Pitchfork Bifurcation - parameter values: $\sigma = 2$, $\alpha = 0.6$, $\mu = 0.8$, $g = CC = 1$ and $\epsilon = 2.75$]

If the extraction productivity is high enough, such that $\tilde{\epsilon} < \epsilon < \bar{\epsilon}$ (supercritical bifurcation), and the transport cost is $\phi^H < \phi < \phi^B$, agglomeration equilibria are stable. In this case, dispersion forces are stronger, so ϕ^S and ϕ^B are lower than in the subcritical case. As transport costs decrease the equilibrium moves to the right in Figure 3, and a supercritical bifurcation takes place at ϕ^B. The main difference is that for a $\phi \in (\phi^S, \phi^B)$ both agglomeration and dispersion equilibria are now locally unstable while the other two non-symmetric interior equilibria are locally stable. Why does this pattern occur? Dispersion forces are strong, so agglomeration equilibria become unstable at a low value of ϕ. At this point, however, the market size effect is still strong due to high transport costs, so the symmetric solution is also unstable. Meanwhile, the non-symmetric equilibria are stable because, if a new firm decides to move to the most populated region, the high extractive productivity in the resource sector causes a sharp increase in the primary prices and dispersion forces activate. In contrast, if a firm decides to move to the less populated region, this firm will have to pay high transport costs to have access to the larger market, and agglomeration forces are set in motion.

When $\phi < \phi^H$ and $\epsilon > \bar{\epsilon}$, the openness of trade is not high enough to guarantee the stability of the symmetric equilibrium, so this high transport cost triggers an agglomeration process. However, the population growth, together with a high extraction productivity (high value of ϵ), accelerate the depletion of the natural resource. The resource dynamics boosts the dispersion forces, first by slowing down the migration flow, and finally reversing it; all of which give rise to a circular behavior. This is consistent with Robinson et al. (2008) who, in a different framework, find that the spatial characteristic of the extraction of a renewable resource ultimately results in cyclical dynamic extraction.

What we find with a renewable and extractable resource is that households move to the region with the higher real wages, and as the market gets bigger, the agglomeration of persons and firms accelerates, so raising the demand for primary goods. The primary sector extracts more natural resources to cope with the increase in demand, compromising its long-term stock, and the level of future extractions. Ultimately, the scarcity of the resource raises the primary prices enough to reverse the migration process. This scheme resembles the chase-and-flee cycle of location of Rauscher (2009), but through a different channel.[9]

The migration flow caused by this circular behavior is compatible with some of the ideas outlined on circular migration in the migration and economic labor literature. Newland (2009) refers to this phenomenon as a seasonal or periodic migration for work, for survival, or as a life-cycle process. Additionally, there have been some attempts to quantify the importance of circular migration and its impact in the origin and the destination countries, see, for example, Constant and Zimmermann (2012); and Agunias and Newland (2007) for other references.

4 A tradable primary good

In this section we present the case of a tradable primary good. To simplify the analysis we assume that the primary good is tradable at the same transport cost of industrial goods, that is, $\nu = \tau$.

4.1 Short-Run Equilibrium

The three markets (labor, industrial and primary goods) clear. Replicating the analysis followed in section 3, it is obtained that (see the online Appendix B for a comprehensive explanation)

$$L_{E_1} w + L_{E_2} = \frac{\mu\left[1+\alpha(\sigma-1)\right]}{\sigma}\left(L_1 w + L_2\right) \tag{47}$$

$$L_{H_1} w + L_{H_2} = \frac{\sigma - \mu\left[1+\alpha(\sigma-1)\right]}{\sigma}\left(L_1 w + L_2\right) \tag{48}$$

$$n_j = \frac{L_{E_j}}{f\left[1+\alpha\left(\sigma-1\right)\right]} \quad \text{for } j = 1, 2. \tag{49}$$

[9] In Rauscher's (2009) chase-and-flee cycle, people prefers a clean and healthy environment, so they decided to stay away from industrial (polluting) activities; but, they are chased by the industries, which want to locate close to the market.

where $w \equiv w_1/w_2$. Moreover, trade between the two regions is balanced if and only if

$$\begin{aligned} TB &= \mu \left(\frac{z_{12}}{1+z_{12}} L_2 - \frac{1}{1+z_{11}} L_1 w \right) + (1-\mu) \left(\frac{q_{12}}{1+q_{12}} L_2 - \frac{1}{1+q_{11}} L_1 w \right) \\ &+ \frac{(1-\alpha)(\sigma-1)}{1+\alpha(\sigma-1)} \left(\frac{q_{12}}{1+q_{12}} L_{E_2} - \frac{1}{1+q_{11}} L_{E_1} w \right) = 0 \end{aligned} \tag{50}$$

where q_{11} is the ratio of region 1 expenditure on local primary good to that on primary good from region 2, and q_{12} is the expenditure of region 2 on region 1 primary good with respect to the primary good from region 2. The first term of equation (50) is the difference between industrial exports and imports of region 1, the second term is the difference between primary exports and imports of region 1 for final consumption, and the third term is the difference between primary (raw material) exports and imports of region 1 to be used as inputs by the industrial firms. Note that if the last two terms of equation (50) vanishes, which is the case if the primary good were not tradable, equation (50) would reduce to (32).

The symmetric equilibrium (39) satisfies equation (50) and the derivative at this point is

$$\frac{\partial TB}{\partial w}(L_1^*, S_1^*, S_2^*, w^*) = \frac{\phi(2\sigma-1+\phi)}{(1+\phi)^2} + \frac{\phi\Psi^*(\phi)}{(1-\phi)^2(1+\phi)} \left[(1+\phi)^2 + 2(\sigma-1)(2\alpha\phi+1-\phi) \right] > 0$$

with $L_1^* = 1/2$, $S_1^* = S_2^* = S^* = \left(1 - \epsilon\frac{\theta}{2}\right) CC$, $w^* = 1$ and $\Psi^*(\phi) > 0$ is (94) evaluated at the symmetric equilibrium. Therefore, for a given value of ϕ, equation (50) implicitly defines w as a function of L_1, S_1 and S_2 in a neighborhood of the symmetric equilibrium.

Using the implicit differentiation in (50), we obtain, at the symmetric equilibrium, that

$$\frac{\partial w}{\partial L_1} = \frac{-4\left[1 - \frac{1+\phi}{1-\phi}\Psi^*(\phi)\right]}{\frac{2\sigma-1+\phi}{1+\phi} + \frac{\Psi^*(\phi)}{(1-\phi)^2}\left[(1+\phi)^2 + 2(\sigma-1)(2\alpha\phi+1-\phi)\right]} \tag{51}$$

which can be negative or positive, depending on the value of ϕ. That is,

$$\frac{\partial w}{\partial L_1} \lesseqgtr 0 \text{ if and only if } \phi \lesseqgtr \hat{\phi} = \frac{\sigma(1-\mu) - \mu[1+\alpha(\sigma-1)]}{\sigma(1-\mu) + \mu[1+\alpha(\sigma-1)]} < 1 \tag{52}$$

In contrast to what happened in Section 3, now if the stock of the natural resources remains constant, the competition effect could be strong enough to dominate the market size effect for high values of transport costs (ϕ low enough).[10]

[10]Note that $\frac{\partial \hat{\phi}}{\partial \alpha} < 0$, which implies that an increase in α reinforces the market size effect. If α increases, the linkages between the two sectors weaken. So, there is a shift of firm expenditures from primary goods (coming form both regions) to labor (a completely local factor). This reinforces the effect of the market size.

Additionally, implicit differentiation in (50) with respect to S_1 and S_2 gives, at the symmetric equilibrium, that

$$\frac{\partial w}{\partial S_1} = -\frac{\partial w}{\partial S_2} = \frac{1}{S^*}\frac{2(\sigma-1)\left[1+\Psi^*(\phi)(1-\alpha)\frac{1+\phi}{1-\phi}\right]}{2\sigma-1+\phi+\frac{\Psi^*(\phi)(1+\phi)}{(1-\phi)^2}\left[(1+\phi)^2+2(\sigma-1)(2\alpha\phi+1-\phi)\right]} > 0 \tag{53}$$

The resource effect is reinforced. The original mechanisms described in Proposition 1 remain, but a new one appears. Note that now a reduction in the primary price due to an increase in S_1 encourages exports of region 1 that must be compensated with an increase in nominal wages of this region. All these mechanisms go in the same direction.

4.2 Long-Run Equilibrium

In the long-run the stock of natural resources does not remain constant; its temporary evolution obeys the differential equation (19) and population migrates from one region to the other according to (21). For the case of a tradable primary good, the ratio of indirect utilities is

$$\frac{V_1}{V_2} = \frac{w_1}{w_2}\left(\frac{P_1}{P_2}\right)^{-\mu}\left(\frac{P_{H_1}}{P_{H_2}}\right)^{-(1-\mu)} \tag{54}$$

where P_{H_j} is the resource price index for region $j = 1, 2$. From equations (8), its mirror image for region 2, and (54) it is clear that when the ratio S_1/S_2 decreases, the ratio of indirect utilities will diminish; and this result is equivalent to property *(ii)* in Proposition 1.

Hence, the differential equations system (19) for $j = 1, 2$ and (21) now takes the form:

$$\dot{L}_1 = L_1(1-L_1)\left[\Delta(w, S_1, S_2) - 1\right] \tag{55}$$

$$\dot{S}_1 = S_1\left[g\left(1-\frac{S_1}{CC}\right) - \epsilon L_{H_1}\right] \tag{56}$$

$$\dot{S}_2 = S_2\left[g\left(1-\frac{S_2}{CC}\right) - \epsilon L_{H_2}\right] = S_2\left[g\left(1\text{-}\frac{S_2}{CC}\right)\text{-}\epsilon\left[\left(g\theta L_1\text{-}L_{H_1}\right)w + g\theta(1\text{-}L_1)\right]\right] \tag{57}$$

with $\Delta(w, S_1, S_2)$ defined in the online Appendix B (see equation (95)), $L_{H_1} = L_1\text{-}L_{E_1}$, $L_{H_2} = \left(g\theta L_1\text{-}L_{H_1}\right)w + g\theta(1\text{-}L_1)$ according to equation (48) and w defined by the balanced trade equation (50) as a function of L_1, S_1 and S_2.

The three steady states defined in (39)-(41) are also steady states of the new system (55)-(57). However, the stability pattern of the symmetric equilibrium may differ from the case of a non-tradable primary good.

4.3 Stability properties

The shaded region in the examples of Figure 4 represent the stability regions of the symmetric equilibrium in the space (ϵ,ϕ) for the different sets of parameters:

[Figure 4 - Stability region of the symmetric equilibrium (tradable primary good)

(a) $\sigma = 7$, $\alpha = 0.5$, $\mu = 0.5$, $g = 0.5$, $CC = 1$; (b) $\sigma = 7$, $\alpha = 0.2$, $\mu = 0.5$, $g = 0.5$, $CC = 1$; (c) $\sigma = 7$, $\alpha = 0.5$, $\mu = 0.5$, $g = 1$, $CC = 1$; and (d) $\sigma = 7$, $\alpha = 0.5$, $\mu = 0.2$, $g = 0.5$, $CC = 1$]

Note that, depending on the value of the parameters, several bifurcation patterns may appear. From Figures 4a - 4d, the predominant pattern for the symmetric equilibrium is the one that goes from stable to unstable as transport costs decrease. Additionally, some regularities are observed and are worthy of mention.

First, when transport cost are very low, the symmetric equilibrium is unstable for all values of $\epsilon \in (0, 2/\theta)$. Because the primary good now can be exported to the other region (at a low transport cost), the advantage of having a lower primary price is limited. Second, when ϵ is low, the symmetric equilibrium is also unstable. This is due to the interaction between the tradability of the primary good and a low resource effect, caused by a low extractive productivity. Third, in the lower-right quadrant in the (ϵ, ϕ) space, transport costs are high and the tradability of the primary good is limited, then, as happened in Section 3, the symmetric equilibrium is unstable.

Finally, if the transport costs of the primary goods were different from those of the industrial goods, similar results could be obtained, but the interaction between the agglomeration and dispersion forces would depend on how these two transport costs relate and vary.

5 Conclusions

This paper presents an extension of Krugman's CP model (1991) and attempts to provide a more comprehensive modelization of the traditional sector, usually treated as residual. Our results allow a better understanding of the migratory processes observed in resource based economies. The model incorporates two key features of the agricultural sector: the dynamics of the renewable natural resources, and the possibility of using raw materials as inputs in the industrial production. In Section 3, it is assumed that the primary good is not directly tradable between regions, in order to isolate the resource effect that arises in the model. In Section 4 we extend the analysis to the case of a tradable primary good. Another major difference between our model and the original CP model is the free labor mobility between sectors.

The core-periphery model presented in this paper has all the effects of the traditional NEG models: market size effect, price index effect and competition effect. Once we incorporate the dynamics of the natural resources into the analysis, a new dispersion force arises: the resource effect. Under certain conditions, this dispersion force overcomes the agglomeration ones driven by the industrial price index and the market size effect, making the symmetric equilibrium stable. In real examples worldwide, it is observed that this force provokes environmental-induced migration (Andrew, 2003; Kirby, 2004; Jouanjean et al., 2014, among others).

If the primary good is not tradable, the effect of transport costs on the stability pattern of the traditional core-periphery models is reversed. For high transport costs one might expect agglomeration to take place (if the new dispersion force is not too strong). However, as transport cost decreases, imports become cheaper and the advantage of being in the largest region diminishes. For example, the construction of new roads increases the profitability of the exploitation of forest and soil in distant areas in Laos, the Philippines, Paraguay, Brazil, Peru, Panama and Bolivia, encouraging the expansion and dispersion of the economic activity (Laurence et al., 2009; and Reymondin et al., 2013).

Our model also gives insights into the transition between agglomeration and dispersion of the economic activity and highlights the role of the extraction productivity in the primary sector. The conditions for a pitchfork bifurcation and a Hopf bifurcation are determined. Depending on the productivity of the primary sector, the pitchfork bifurcation can be subcritical or supercritical, and these two patterns illustrate different processes. On the one hand, strong agglomeration forces (subcritical), imply a sudden change in the spatial distribution of the economic activity, as in the rapid shift that took place in the exploitation of the sea urchin fisheries in Chile (Andrew et al., 2003). On the other hand, strong dispersion forces (supercritical), imply a smooth change, as observed in the slow depopulation driven by the decline in soil fertility in Italy (Dazzi and Lo papa, 2013)

Another important result in our paper is the existence of a Hopf bifurcation, which makes the appearance of a branch of periodic solutions feasible, so introducing cyclic behavior in the dynamics. When the extraction productivity of the primary sector is too high , economic activity will tend to agglomerate in one region until the primary good becomes too expensive, and then a dispersion process takes place. However, due to the high extraction productivity, the stock of the resource takes longer to renew and, while this happens, more firms continue to arrive in the other region, so agglomeration is taking place now in this region. This is a completely new result in CP models in continuous-time.

If the primary good is tradable, several bifurcation patterns may appear, depending on the value of the parameters. Also, some regularities are observed. First, reductions in the transport costs of the primary goods weaken the dispersion forces associated to the resource, then, for low values of transport cost the dispersion equilibrium is unstable. Second, for low values of the extraction productivity, the symmetric equilibrium is also unstable. In most cases, the symmetric equilibrium goes from stable to unstable as the openness of trade increases.

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Appendix A

Proof of equation (32): Following Krugman (1991), $z_{11} \equiv \frac{n_1 c_{11} p_1}{n_2 c_{21} p_2 \tau}$ can be defined as the ratio of region 1 expenditure on local manufactures to that on manufactures from region 2. In a similar way, $z_{12} \equiv \frac{n_1 c_{12} p_1 \tau}{n_2 c_{22} p_2}$ is the expenditure of region 2 on region 1 industrial goods with respect to goods produced in region 2. Thus, the equilibria for the industrial sectors in both regions are

$$n_1 p_{H_1} h_1 + w_1 L_{E_1} = \mu \left\{ \frac{z_{11}}{1+z_{11}} L_1 w_1 + \frac{z_{12}}{1+z_{12}} L_2 w_2 \right\} \tag{58}$$

$$n_2 p_{H_2} h_2 + w_2 L_{E_2} = \mu \left\{ \frac{1}{1+z_{11}} L_1 w_1 + \frac{1}{1+z_{12}} L_2 w_2 \right\} \tag{59}$$

Using equations (26), (30) and (31), the previous two equations can be reduced to the single

$$TB = \overbrace{\frac{1}{1+z_{11}} L_1 w_1}^{\text{Imports of region 1}} - \overbrace{\frac{z_{12}}{1+z_{12}} L_2 w_2}^{\text{Exports of region 1}} = 0 \tag{60}$$

which guarantees that trade is balanced. Rearranging terms in equation (60) we have that

$$z_{12} \left(1 + z_{11}\right) \frac{L_2}{L_1} - \left(1 + z_{12}\right) w = 0 \tag{61}$$

where $w \equiv w_1 / w_2$. Note that, from (31) and (5),

$$z_{11} \equiv \frac{n_1 c_{11} p_1}{n_2 c_{21} p_2 \tau} = \frac{L_1}{L_2} \left(\frac{p}{\tau}\right)^{1-\sigma} \quad \text{and} \quad z_{12} \equiv \frac{n_1 c_{11} p_1}{n_2 c_{21} p_2 \tau} = \frac{L_1}{L_2} \left(p\tau\right)^{1-\sigma} \tag{62}$$

where $p = p_1 / p_2$ is defined in (24). Replacing (62) into (61), and taking into account (24), equation (32) is obtained.

Moreover, given L_1, S_1, S_2 and ϕ, function $p(S_1/S_2)^{1-\alpha}$ is an increasing straight line as a function of p (which takes the value 0 at $p = 0$) and $p^{1-\sigma}(\phi + L_1/(1-L_1)p^{1-\sigma})/(1 + L_1/(1-L_1)p^{1-\sigma}\phi)$ is a strictly increasing and convex function of p. Then, given the values of L_1, S_1, S_2 and ϕ, there exists a unique positive value p such that (32) is satisfied.

Proof of proposition 1: From equations (24) and (32) we have that

$$\hat{p} = \frac{\left(np^{1-\sigma}\right) \hat{n} - (1-\alpha)\psi_1 \left(\hat{S}_1 - \hat{S}_2\right)}{\psi_1 + \psi_2} \tag{63}$$

$$\psi_1 \equiv \left(w/p^{1-\sigma}\right) \left(1 + np^{1-\sigma}\right)^2 > 0 \tag{64}$$

$$\psi_2 \equiv (\sigma - 1)\left(1 + np^{1-\sigma}\right) \left\{ \left[\phi \left(1 + \left(np^{1-\sigma}\right)^2\right) + 2np^{1-\sigma} \right] \right\} > 0 \tag{65}$$

where $n \equiv n_1/n_2$, $w \equiv w_1/w_2$, and $\hat{x} \equiv dx/x$ for each variable x. Additionally, from (24) we have that

$$\hat{w} = \hat{p} + (1-\alpha)\left(\hat{S}_1 - \hat{S}_2\right)$$

Then, by using expression (63) we obtain

$$\hat{w} = \frac{\left(np^{1-\sigma}\right)\hat{n} + (1-\alpha)\,\psi_2\left(\hat{S}_1 - \hat{S}_2\right)}{\psi_1 + \psi_2} \tag{66}$$

Moreover, from (7), (10) and (24), we have that

$$\begin{aligned}\hat{P} &= \frac{np^{1-\sigma}\left(1-\phi^2\right)}{\left(np^{1-\sigma}+\phi\right)\left(np^{1-\sigma}\phi+1\right)}\left(\hat{p} - \frac{\hat{n}}{\sigma-1}\right)\\ \hat{p}_H &= \hat{p} - \alpha\left(\hat{S}_1 - \hat{S}_2\right)\end{aligned}$$

where $P \equiv P_1/P_2$ and $p_H \equiv p_{H_1}/p_{H_2}$. Replacing expression (63) we arrive to

$$\hat{P} = -\frac{1-\phi^2}{\left(np^{1-\sigma}\right)^{-1}}\frac{\left[\psi_1 - (\sigma-1)\,np^{1-\sigma} + \psi_2\right]\hat{n} + (1-\alpha)(\sigma-1)\,\psi_1\left(\hat{S}_1 - \hat{S}_2\right)}{(\sigma-1)\left(np^{1-\sigma}+\phi\right)\left(np^{1-\sigma}\phi+1\right)(\psi_1+\psi_2)} \tag{67}$$

$$\hat{p}_H = \frac{\left(np^{1-\sigma}\right)\hat{n} - (\psi_1 + \alpha\psi_2)\left(\hat{S}_1 - \hat{S}_2\right)}{\psi_1 + \psi_2} \tag{68}$$

where $\psi_1 - (\sigma-1)\,np^{1-\sigma} > 0$. In the three expressions (66)-(68), the resource effect are the terms associated to $\left(\hat{S}_1 - \hat{S}_2\right)$. Note that, if population increases in region 1, the steady state values S_j^*, given by (33), vary

$$\frac{dS_1^*}{dL_1} \leq 0, \quad \frac{dS_2^*}{dL_1} \geq 0$$

Since S_j^*, $j = 1, 2$, are globally stable, the natural resources S_j, $j = 1, 2$, will adjust immediately to their long-run values. Thus, $\hat{S}_1 - \hat{S}_2 < 0$, which implies that, due to the resource dynamics, the ratio of nominal wages decreases (property (i)), according to (66). Additionally, the ratio of industrial price indexes and the ratio of primary prices increases (property (ii)), according to (67) and (68) respectively.

Obviously, while the stock of natural resources moves to its long-run level S_j^*, the remaining variables of the model move simultaneously. The final effect on the indirect utilities will be the addition of the previous effect, linked to the resource, and the usual ones: competition, market size and price index effects.

Proof of proposition 2: The Jacobian matrix of the dynamic system (35)-(37) at the symmetric solution (39) is

$$J^*_{1/2} = \begin{pmatrix} a & b & -b \\ -c & d & 0 \\ c & 0 & d \end{pmatrix} \tag{69}$$

with

$$a=\frac{\mu(2\sigma-1)}{\sigma-1}\frac{(1-\phi)}{2\sigma-(1-\phi)},\ b=\frac{1}{4S^*}\left(1-\mu+\mu(1-\alpha)\frac{2\sigma-(1+\phi)}{2\sigma-(1-\phi)}\right),\ c=\epsilon\theta g S^*,\ d=-\frac{gS^*}{CC}. \tag{70}$$

where $S^* = (1\text{-}\epsilon\theta/2)CC$.The characteristic polynomial is equal to $P(\lambda)=(d\text{-}\lambda)[\lambda^2\text{-}(a+d)\lambda+ad+2bc]$, so the eigenvalues can be explicitly calculated and the three of them are negative if and only if (43) is satisfied.

In the case of the agglomeration equilibrium (40), the Jacobian matrix is

$$J^*_1 = \begin{pmatrix} 1-(1-\epsilon\theta)^{1-\mu\alpha-\mu(1-\alpha)/\sigma}\,\phi^{-\frac{\mu(2\sigma-1)}{\sigma(\sigma-1)}} & 0 & 0 \\ -\epsilon\theta g S^*_1 & -g\,(1-\epsilon\theta) & 0 \\ \epsilon\theta g S^*_2 & 0 & -g \end{pmatrix}.$$

Hence, the three eigenvalues are negative if and only if condition (44) is satisfied. The same condition ensures the stability of the equilibrium (41).

The case of agglomeration equilibrium (42) though, can not be analyzed through the Jacobian matrix. Nevertheless, it is always unstable. Note that if $S_1 -> CC$ and $S_2 \to 0$, a solution with L_1 diminishing while $p -> 0$ is not reachable and (42) is unstable.

Proposition 3[11] If $\phi^B, \phi^S \in (0,1)$ and $\epsilon < \bar{\epsilon}$, there exist two interior non-symmetric equilibria that bifurcate from the symmetric equilibrium at the value $\phi = \phi^B$. At this point, the stability properties of the symmetric equilibrium change and a pitchfork bifurcation appears. If $\epsilon\theta < 1$, the two branches of the bifurcation (new equilibria) converge to the agglomeration equilibria at $\phi = \phi^S$. Furthermore,

(i) if $\epsilon < \min\{\tilde{\epsilon}, \bar{\epsilon}\}$, the pitchfork bifurcation is subcritical; that is, the equilibria on the branches are locally unstable.

[11] This proposition has been stated in section 3.3 in economic terms. Here, in order to follow the proof, we have restated it using more technical language.

(ii) if $\tilde{\epsilon} < \epsilon < \bar{\epsilon}$ the pitchfork bifurcation is supercritical; that is, the equilibria on the branches are locally stable.

where $\tilde{\epsilon} > 0$ is the intersection point of ϕ^B and ϕ^S.

Proof of proposition 3: It is necessary to prove the existence of the non-symmetric interior equilibria and only two branches of non-symmetric interior equilibria exist. To prove their existence, we look for a pitchfork bifurcation of the symmetric equilibrium, following Guckenheimer and Holmes (1983) and Forslid and Ottaviano (2003). Note that a pitchfork bifurcation only takes place when $\epsilon < \bar{\epsilon}$. Otherwise, if $\epsilon > \bar{\epsilon}$, the curve ϕ^B separates two regions for which the symmetric equilibrium is unstable (a saddle point vs. an unstable node).

The following steps are taken: 1. The variables are changed so that the system has a fixed point at the origin $(0, 0, 0)$; 2. A new parameter (γ) is defined such that for $\gamma = 0$ the Jacobian matrix has an eigenvalue equal to zero; 3. A change of coordinates is made using the eigenvectors; 4. A Taylor second order approximation of the center manifold is made; 5. The derivatives that prove the existence of a pitchfork bifurcation are calculated; and 6. The sign of the derivatives is analyzed, which determines if the bifurcation is subcritical or supercritical

Step 1: Note that $\left|J^*_{1/2}\right| = 0$ if and only if $\phi = \phi^B$, then the symmetric equilibrium (39) becomes non-hyperbolic at $\phi = \phi^B$ and it is characterized by a one-dimensional center manifold. For mathematical tractability, variables L_1, S_1 and S_2 are changed to $l = L_1 - 1/2$, $s_1 = S_1 - CC\left[1 - \epsilon\theta/2\right]$ and $s_2 = S_2 - CC\left[1 - \epsilon\theta/2\right]$, so the new system would have a fixed point at the origin $(0, 0, 0)$.

Step 2: We define a new parameter $\gamma \equiv \mu(2\sigma - 1)\left(\frac{1-\phi}{2\sigma-(1-\phi)} - \frac{1-\phi^B}{2\sigma-(1-\phi^B)}\right)$, so $\gamma = 0$ if and only if $\phi = \phi^B$. If this is the case, we shall call the Jacobian matrix $J^*_{(1/2,0)}$ and, for the cases where $\gamma \neq 0$, this matrix will be called $J^*_{(1/2,\gamma)}$. The dynamics of l, s_1 and s_2 are given by

$$\begin{pmatrix} \dot{l} \\ \dot{s}_1 \\ \dot{s}_2 \end{pmatrix} = J^*_{\left(\frac{1}{2},\gamma\right)} \begin{pmatrix} l \\ s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} g^l(l, s_1, s_2) \\ g^{s_1}(l, s_1, s_2) \\ g^{s_2}(l, s_1, s_2) \end{pmatrix}$$

where $g^l(l, s_1, s_2)$, $g^{s_1}(l, s_1, s_2)$ and $g^{s_2}(l, s_1, s_2)$ form the non-linear part of the system.

Note that the following property is satisfied:

$$J^*_{(\frac{1}{2},\gamma)} = J^*_{(\frac{1}{2},0)} + \gamma \begin{pmatrix} \frac{1}{\sigma-1} & \frac{1-\alpha}{4\sigma S^*} & -\frac{1-\alpha}{4\sigma S^*} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

where $S^* = (1-\epsilon\theta/2)CC$. The matrix $J^*_{(1/2,0)}$ has the following eigenvalues: 0,-g(1-$\epsilon\frac{\theta}{2}$), and $\mu(2\sigma-1)(1-\phi^B)/\left[(\sigma-1)(2\sigma-(1-\phi^B))\right] - g(1-\epsilon\frac{\theta}{2})$.

Step 3: Using the eigenvectors as a basis for a new coordinate system (u, v, and w), we set

$$\begin{pmatrix} l \\ s_1 \\ s_2 \end{pmatrix} = Q \begin{pmatrix} u \\ v \\ w \end{pmatrix} \text{ with } Q = \begin{pmatrix} \frac{1}{\epsilon\theta CC} & 0 & \frac{2\sigma(1-\alpha\mu)-\mu(1-\alpha)}{gCC(2-\epsilon\theta)[\epsilon\theta(1+\alpha(\sigma-1))+2\sigma(1-\epsilon\theta)]} \\ -1 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix},$$

where Q is the matrix of eigenvectors of $J^*_{(1/2,0)}$. Then,

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = Q^{-1} J^*_{(1/2,0)} Q \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \gamma Q^{-1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} Q \begin{pmatrix} u \\ v \\ w \end{pmatrix} + Q^{-1} \begin{pmatrix} g^l((u,v,w)^t Q^t) \\ g^{s_1}((u,v,w)^t Q^t) \\ g^{s_2}((u,v,w)^t Q^t) \end{pmatrix} \tag{71}$$

Step 4: Let $v = h_1(u,\gamma) = a_1u^2 + b_1u\gamma + c_1\gamma^2$ and $w = h_2(u,\gamma) = a_2u^2 + b_2u\gamma + c_2\gamma^2$ be the second order Taylor approximation of the invariant center manifold . Taking this into account in (71) we obtain that $\dot{u} = f(u,\gamma) + \mathcal{O}(3)$, where $\mathcal{O}(3)$ means terms of order u^3, $u^2\gamma$, $u\gamma^2$ and γ^3.

Moreover,

$$\dot{v} = \left(\frac{\partial h_1}{\partial u}, \frac{\partial h_1}{\partial \gamma}\right) \begin{pmatrix} \dot{u} \\ \dot{\gamma} \end{pmatrix} = (2a_1u + b_1\gamma)\,\dot{u} + (b_1u + 2c_1\gamma)\,\dot{\gamma} \tag{72a}$$

$$\dot{w} = \left(\frac{\partial h_2}{\partial u}, \frac{\partial h_2}{\partial \gamma}\right) \begin{pmatrix} \dot{u} \\ \dot{\gamma} \end{pmatrix} = (2a_2u + b_2\gamma)\,\dot{u} + (b_2u + 2c_2\gamma)\,\dot{\gamma} \tag{72b}$$

$$\dot{\gamma} = 0 \tag{72c}$$

Step 5: We can directly calculate $\frac{\partial \dot{u}}{\partial u}$, $\frac{\partial^3 \dot{u}}{\partial u^3}$, and $\frac{\partial^2 \dot{u}}{\partial u \partial \gamma}$ in the center manifold for ($u = 0$ and $\gamma = 0$) by using expressions (72a), (72b), and $\dot{u}$ from the system (71). For calculating these derivatives we have used the Taylor polynomial of order three of p, implicitly defined by (32) as a function of S_1/S_2 and L_1, at the symmetric equilibrium.

$$\frac{\partial \dot{u}}{\partial u}(0,\gamma) = \frac{\partial f}{\partial u}(0,\gamma) = 0 \tag{73}$$

$$\frac{\partial^2 \dot{u}}{\partial u \partial \gamma}(0,0) < 0 \tag{74}$$

These results together indicate a pitchfork bifurcation. The first derivative (73) implies that $u = 0$ is always an equilibrium, and that $\dot{u}$ rotates above this equilibrium. The cross derivative (74) shows in which direction the equilibrium loses its stability. From the analysis of $J^*_{\frac{1}{2}}$ in proposition 2 and the definition of γ, it is known that the equilibrium $u = 0$ is stable when $\gamma < 0$, and unstable when $\gamma > 0$. Then, the cross derivative (74) is negative.

Note that ϕ^B and ϕ^S take the value 1 at $\epsilon = 0$ and both curves decreace as ϵ increases. At $\epsilon = \theta^{-1}$ function $\phi^B > \phi^S = 0$. Moreover, $\partial^2\phi^S/\partial\epsilon^2 > \partial^2\phi^B/\partial\epsilon^2$ at $\epsilon = 0$, therefore, there exists a value $0 < \tilde{\epsilon} < \theta^{-1}$ such that $\phi^B < \phi^S$ if $\epsilon < \tilde{\epsilon}$ and $\phi^B > \phi^S$ if $\epsilon > \tilde{\epsilon}$. At $\tilde{\epsilon}$ the sign of the following derivative changes.

$$\frac{\partial^3 \dot{u}}{\partial u^3}(0,0) = \frac{16[\sigma\text{-}\mu(1+\alpha(\sigma\text{-}1))]}{\mu^3 CC^3 \left(1\text{-}\frac{\epsilon\theta}{2}\right)^3} \frac{A\,(\epsilon\theta)^2 + B\,\epsilon\theta + C}{\det Q\,(2\sigma\text{-}1)^2\epsilon\theta[\epsilon\theta(1+\alpha(\sigma\text{-}1)) + 2\sigma(1\text{-}\epsilon\theta)]} \tag{75}$$

where $C = 12\mu^2(\sigma\text{-}1)(2\sigma\text{-}1)(1\text{-}\mu\text{-}\sigma(1\text{-}\mu\alpha)) < 0$, A and B are constants that depend on σ, μ and α.

The sign of the third derivative (75) indicates if the bifurcation is subcritical or supercritical. This issue will be studied in step 6.

Once the existence of two non-symmetric equilibria is granted, it is necessary to prove that there are only two branches of interior non-symmetric equilibria. According to equation (35), the non-symmetric interior equilibria should satisfy the following equation for $L_1 \neq \frac{1}{2}$,

$$p = \left[\frac{1 - \epsilon\theta L_1}{1 - \epsilon\theta\,(1 - L_1)}\right]^{-\rho} \text{ with } \rho \equiv \frac{(1 - \alpha\mu)\,(\sigma - 1) + \mu\,(1 - \alpha)}{\mu\,(2\sigma - 1)} \tag{76}$$

where p is a function of L_1 and ϕ defined by equation (32), that can be restate as,

$$\phi\,(L_1, S_1, S_2, p) = p^{-(1-\sigma)} \frac{(1 - L_1)\,p\,(S_1/S_2)^{1-\alpha} - L_1 p^{2(1-\sigma)}}{(1 - L_1) - L_1 p\,(S_1/S_2)^{1-\alpha}} \tag{77}$$

Substituting (77) into (76) we obtain a map that assigns a unique value of ϕ for each value of L_1, as depicted in Figure 2. Indeed, equation (76) defines ϕ as a continuous and differentiable function of L_1, if $L_1 \neq \frac{1}{2}$,

$$\phi(L_1) = \left[\frac{1-\epsilon\theta L_1}{1-\epsilon\theta(1-L_1)}\right]^{-\rho(\sigma-1)} \frac{(1-L_1)\left[\frac{1-\epsilon\theta L_1}{1-\epsilon\theta(1-L_1)}\right]^{1-\alpha-\rho} - L_1\left[\frac{1-\epsilon\theta L_1}{1-\epsilon\theta(1-L_1)}\right]^{2\rho(\sigma-1)}}{(1-L_1) - L_1\left[\frac{1-\epsilon\theta L_1}{1-\epsilon\theta(1-L_1)}\right]^{1-\alpha-\rho}} \tag{78}$$

Note that $\phi(0) = \phi(1) = (1-\epsilon\theta)^{\frac{\sigma-1}{\mu(2\sigma-1)}[\sigma(1-\mu\alpha)-\mu(1-\alpha)]} = \phi^S$. This implies that the non-symmetric equilibria emerging from the bifurcation and the ones characterized by equation (76) form two branches that are born at ϕ^B and converge to ϕ^S if the economy agglomerates ($L_1 = 0$ or $L_1 = 1$).

Step 6. The sign of the third derivative $\frac{\partial^3 \dot{u}}{\partial u^3}(0,0)$ indicates whether the bifurcation is subcritical or supercritical. The denominator in (75) is positive. Therefore, the third derivative is negative (positive) if $\epsilon < \tilde{\epsilon}$ ($\tilde{\epsilon} < \epsilon < \bar{\epsilon}$), predicting a subcritical pitchfork bifurcation (supercritical).

Proposition 4[12] If $\phi > \phi^i$, the critical value ϕ^H is a Hopf bifurcation point of system (35)-(37).

Proof of proposition 4: The eigenvalues of (69) are

$$\lambda_{1,2} = \frac{(a+d) \pm \sqrt{(a+d)^2 - 4(ad+2bc)}}{2a} \text{ and } \lambda_3 = d < 0$$

with a, b, c and d defined in (70).

Let us define a new parameter $\eta \equiv \phi - \phi^H = -\frac{2\sigma-1+\phi}{\frac{\mu(2\sigma-1)}{\sigma-1}+g(1-\epsilon\frac{\theta}{2})}(a+d)$. Note that $\eta = 0$ if and only if $\phi = \phi^H$ and, if this is the case, $\lambda_{1,2}$ are two conjugate eigenvalues with zero real part.

According to Gandolfo (1997, page 477), the system has a family of periodic solutions if

(i) it possesses a pair of simple complex conjugate eigenvalues $\theta(\eta) \pm \omega(\eta)i$, that become pure imaginary at the critical value η_0, and no other eigenvalues with zero real part exist

[12] This proposition has been stated in section 3.3 in economic terms. Here, in order to follow the proof, we have restated it using more technical language.

(ii) and

$$\left.\frac{\partial\theta(\eta)}{\partial\eta}\right|_{\eta_0} \neq 0$$

Eigenvalues $\lambda_{1,2}$ are simple complex conjugate if $(a+d)^2 < 4\,(ad+2bc)$, that is

$$\left(\frac{\mu(2\sigma-1)}{\sigma-1}(\delta-\delta^H)\right)^2 < 4\mu(2\sigma-1)g\left(\frac{1-\alpha}{\sigma}\frac{\epsilon\theta}{2}-\frac{1-\epsilon\theta/2}{\sigma-1}\right)(\delta-\delta^B) \tag{79}$$

where $\delta=(1-\phi)/(2\sigma-(1-\phi))$, $\delta^H=(1-\phi^H)/(2\sigma-(1-\phi^H))$ and $\delta^B=(1-\phi^B)/(2\sigma-(1-\phi^B))$ which is equivalent to condition

$$\delta < \delta^i \tag{80}$$

where δ^i is the solution of the quadratic equation $(a+d)^2-4\,(ad+2bc)=0$ that satisfies $\delta^i \leq \delta^B$. Note that if $\delta^H=\delta^B$ then $\delta^i=\delta^H=\delta^B$. The previous condition is equivalent to $\phi>\phi^i$.

Moreover, if $\eta_0 = 0$ condition (i) is satisfied. Additionally, given that $a+d$ =-$\eta(\frac{\mu}{\sigma-1}(2\sigma\text{-}1)+\frac{gS^*}{CC})/(2\sigma\text{-}1+\phi)$, condition (ii) is also satisfied. Then, $\eta_0 = 0$ is a Hopf bifurcation point.

Appendix B

Short-Run Equilibrium

Equilibrium in the industrial sectors. We define

$$z_{11} \equiv \frac{n_1 c_{11} p_1}{n_2 c_{21} p_2 \tau} = \left(\frac{p}{\tau}\right)^{1-\sigma}\frac{L_{E_1}}{L_{E_2}} \quad\text{and}\quad z_{12} \equiv \frac{n_1 c_{12} p_1 \tau}{n_2 c_{22} p_2} = (p\tau)^{1-\sigma}\frac{L_{E_1}}{L_{E_2}} \tag{81}$$

where $p=p_1/p_2$. Thus, the equilibrium for the industrial sectors in both regions requires

$$n_1\overbrace{(p_{H_1}h_{11}+\tau p_{H_2}h_{12})}^{\frac{1-\alpha}{\alpha}w_1(l_{x_1}-f)}+w_1L_{E_1} = \mu\left\{\frac{z_{11}}{1+z_{11}}L_1w_1+\frac{z_{12}}{1+z_{12}}L_2w_2\right\} \tag{82}$$

$$n_2\underbrace{(\tau p_{H_1}h_{21}+p_{H_2}h_{22})}_{\frac{1-\alpha}{\alpha}w_2(l_{x_2}-f)}+w_2L_{E_2} = \mu\left\{\frac{1}{1+z_{11}}L_1w_1+\frac{1}{1+z_{12}}L_2w_2\right\} \tag{83}$$

Taking into account (18) and industrial demands for primary goods h_{jk} in (14)-(15) the previous system of equations transforms into

$$\frac{\sigma}{1+\alpha(\sigma-1)}L_{E_1}w_1 = \mu\left\{\frac{z_{11}}{1+z_{11}}L_1w_1+\frac{z_{12}}{1+z_{12}}L_2w_2\right\} \quad (84)$$

$$\frac{\sigma}{1+\alpha(\sigma-1)}L_{E_2}w_2 = \mu\left\{\frac{1}{1+z_{11}}L_1w_1+\frac{1}{1+z_{12}}L_2w_2\right\} \quad (85)$$

Dividing by salaries w_j, adding both equations and taking into account that $L_1+L_2=1$ we obtain (47) and (48). Moreover,

$$L_{E_1}=\frac{\mu\left[1+\alpha(\sigma-1)\right]}{\sigma}\left(L_1w+L_2\right)\frac{\lambda}{1+\lambda w} \quad \text{and} \quad L_{E_2}=\frac{L_{E_1}}{\lambda} \quad (86)$$

with

$$\lambda\equiv\frac{L_{E_1}}{L_{E_2}}=p^{-(1-\sigma)}\frac{\frac{\phi}{p^{1-\sigma}/w-\phi}-\frac{1}{1-\phi p^{1-\sigma}/w}\frac{L_1w}{L_2}}{\frac{\phi}{1-\phi p^{1-\sigma}/w}\frac{L_1w}{L_2}-\frac{1}{p^{1-\sigma}/w-\phi}}$$

Equilibrium in the primary good sectors. The equilibrium in the primary sector requires that harvesting equals the demand of primary good from consumers and from the industrial sector. That is,

$$\begin{aligned} H_1 = {} & L_1\frac{(1-\mu)\,w_1}{P_{H_1}}\left(\frac{p_{H_1}}{P_{H_1}}\right)^{-\sigma}+n_1\frac{1-\alpha}{\alpha}\frac{w_1}{P_{H_1}}\left(l_{x_1}-f\right)\left(\frac{p_{H_1}}{P_{H_1}}\right)^{-\sigma} \\ & +L_2\tau\frac{(1-\mu)\,w_2}{P_{H_2}}\left(\frac{\tau p_{H_1}}{P_{H_2}}\right)^{-\sigma}+n_2\tau\frac{1-\alpha}{\alpha}\frac{w_2}{P_{H_2}}\left(l_{x_2}-f\right)\left(\frac{\tau p_{H_1}}{P_{H_2}}\right)^{-\sigma} \end{aligned} \quad (87)$$

$$\begin{aligned} H_2 = {} & L_2\frac{(1-\mu)\,w_2}{P_{H_2}}\left(\frac{p_{H_2}}{P_{H_2}}\right)^{-\sigma}+n_2\frac{1-\alpha}{\alpha}\frac{w_2}{P_{H_2}}\left(l_{x_2}-f\right)\left(\frac{p_{H_2}}{P_{H_2}}\right)^{-\sigma} \\ & +L_1\tau\frac{(1-\mu)\,w_1}{P_{H_1}}\left(\frac{\tau p_{H_2}}{P_{H_1}}\right)^{-\sigma}+n_1\tau\frac{1-\alpha}{\alpha}\frac{w_1}{P_{H_1}}\left(l_{x_1}-f\right)\left(\frac{\tau p_{H_2}}{P_{H_1}}\right)^{-\sigma} \end{aligned} \quad (88)$$

Moreover, from (87)-(88), taking into account (9), (10) and (18) it is obtained that

$$L_{H_1}w_1 = \frac{q_{11}}{1+q_{11}}\left\{(1\text{-}\mu)L_1+\frac{(1\text{-}\alpha)(\sigma\text{-}1)}{1+\alpha(\sigma\text{-}1)}L_{E_1}\right\}w_1+\frac{q_{12}}{1+q_{12}}\left\{(1\text{-}\mu)L_2+\frac{(1\text{-}\alpha)(\sigma\text{-}1)}{1+\alpha(\sigma\text{-}1)}L_{E_2}\right\}w_2 \quad (89)$$

$$L_{H_2}w_2 = \frac{1}{1+q_{11}}\left\{(1\text{-}\mu)L_1+\frac{(1\text{-}\alpha)(\sigma\text{-}1)}{1+\alpha(\sigma\text{-}1)}L_{E_1}\right\}w_1+\frac{1}{1+q_{12}}\left\{(1\text{-}\mu)L_2+\frac{(1\text{-}\alpha)(\sigma\text{-}1)}{1+\alpha(\sigma\text{-}1)}L_{E_2}\right\}w_2 \quad (90)$$

where the following definitions should be taken into account:

$$q_{11} \equiv \frac{\left(n_1h_{11}+L_1c_{H_{11}}\right)p_{H_1}}{\left(n_1h_{21}+L_1c_{H_{21}}\right)p_{H_2}\tau}=\left(\frac{w}{\tau}\right)^{1-\sigma}\left(\frac{S_1}{S_2}\right)^{-(1-\sigma)} \quad (91)$$

$$q_{12} \equiv \frac{\left(n_2h_{12}+L_2c_{H_{12}}\right)p_{H_1}\tau}{\left(n_2h_{22}+L_2c_{H_{22}}\right)p_{H_2}}=(w\tau)^{1-\sigma}\left(\frac{S_1}{S_2}\right)^{-(1-\sigma)} \quad (92)$$

whose interpretations, for the primary goods, are similar to z_{11} and z_{12}.

Balanced trade equation. Replacing L_{E_1} and L_{E_2} from (84) and (85) into (89) and (90), and taking into account that $q_{11}/(1+q_{11}) = 1 - 1/(1+q_{11})$ we get

$$\mu L_{H_1} w - \left[\frac{\sigma}{1+\alpha(\sigma\text{-}1)} - \mu\right] L_{E_1} w = (1\text{-}\mu)\left(\frac{q_{12}}{1+q_{12}} L_2 - \frac{1}{1+q_{11}} L_1 w\right)$$
$$+\frac{\mu(1\text{-}\alpha)(\sigma\text{-}1)}{\sigma}\left[\frac{q_{12}}{1+q_{12}}\left(\frac{1}{1+z_{11}} L_1 w + \frac{1}{1+z_{12}} L_2\right) - \frac{1}{1+q_{11}}\left(\frac{z_{11}}{1+z_{11}} L_1 w + \frac{z_{12}}{1+z_{12}} L_2\right)\right]$$

Applying that $1/(1+z_{12}) = 1 - z_{12}/(1+z_{12})$ and $z_{11}/(1+z_{11}) = 1 - 1/(1+z_{11})$, together with (47)-(48),

$$\mu L_{H_1} w - \mu \frac{L_{H_1} w + L_{H_2}}{L_{E_1} w + L_{E_2}} L_{E_1} w = (1\text{-}\mu)\left(\frac{q_{12}}{1+q_{12}} L_2 - \frac{1}{1+q_{11}} L_1 w\right)$$
$$+\frac{\mu(1\text{-}\alpha)(\sigma\text{-}1)}{\sigma}\left[\left(\frac{1}{1+z_{11}} L_1 w - \frac{z_{12}}{1+z_{12}} L_2\right)\left(\frac{q_{12}}{1+q_{12}} + \frac{1}{1+q_{11}}\right) + \frac{q_{12}}{1+q_{12}} L_2 - \frac{1}{1+q_{11}} L_1 w\right]$$

Rearranging the terms:

$$\mu\left(L_1 w - \frac{L_{E_1} w}{L_{E_1} w + L_{E_2}}\right) - \frac{\mu(1\text{-}\alpha)(\sigma\text{-}1)}{\sigma}\left(\frac{1}{1+z_{11}} L_1 w - \frac{z_{12}}{1+z_{12}} L_2\right)\left(\frac{q_{12}}{1+q_{12}} + \frac{1}{1+q_{11}}\right) =$$
$$\left[1 - \mu + \frac{\mu(1\text{-}\alpha)(\sigma\text{-}1)}{\sigma}\right]\left(\frac{q_{12}}{1+q_{12}} L_2 - \frac{1}{1+q_{11}} L_1 w\right)$$

Therefore, the following equation is obtained, which guarantees that trade is balanced (imports of region j equals exports of region j),

$$\left(\frac{1}{1+z_{11}}\frac{L_1 w}{L_2} - \frac{z_{12}}{1+z_{12}}\right)\Psi = \left(\frac{q_{12}}{1+q_{12}} - \frac{1}{1+q_{11}}\frac{L_1 w}{L_2}\right) \tag{93}$$

which is equivalent to equation (50). Function Ψ, is given by

$$\Psi = \frac{\mu - \frac{\mu(1-\alpha)(\sigma-1)}{\sigma}\left(\frac{q_{12}}{1+q_{12}} + \frac{1}{1+q_{11}}\right)}{1 - \mu + \frac{\mu(1-\alpha)(\sigma-1)}{\sigma}} \tag{94}$$

Dynamics Population dynamics depends on the ratio of indirect utilities V_j given by

$$\frac{P_1}{P_2} = \left(\frac{n_1 p_1^{1-\sigma} + n_2 (p_2\tau)^{1-\sigma}}{n_1 (p_1\tau)^{1-\sigma} + n_2 p_2^{1-\sigma}}\right)^{\frac{1}{1-\sigma}} = \left(\frac{\lambda p^{1-\sigma} + \phi}{\phi\lambda p^{1-\sigma} + 1}\right)^{\frac{1}{1-\sigma}}$$
$$\frac{P_{H_1}}{P_{H_2}} = \left(\frac{p_{H_1}^{1-\sigma} + (p_{H_2}\tau)^{1-\sigma}}{(p_{H_1}\tau)^{1-\sigma} + p_{H_2}^{1-\sigma}}\right)^{\frac{1}{1-\sigma}} = \left(\frac{w^{1-\sigma}\left(\frac{S_1}{S_2}\right)^{-(1-\sigma)} + \phi}{\phi w^{1-\sigma}\left(\frac{S_1}{S_2}\right)^{-(1-\sigma)} + 1}\right)^{\frac{1}{1-\sigma}}$$

and the dynamic evolutions of the stocks the natural resources and population in the two regions are driven by equations (55)-(57), with

$$\Delta(w, S_1, S_2) = w\left(\frac{\lambda p^{1-\sigma} + \phi}{\phi\lambda p^{1-\sigma} + 1}\right)^{\frac{-\mu}{1-\sigma}}\left(\frac{w^{1-\sigma}(S_1/S_2)^{-(1-\sigma)} + \phi}{\phi w^{1-\sigma}(S_1/S_2)^{-(1-\sigma)} + 1}\right)^{-\frac{1-\mu}{1-\sigma}} \tag{95}$$

where, as in (16)

$$p = w^{\alpha}\left(\frac{w^{1-\sigma}(S_1/S_2)^{-(1-\sigma)}+\phi}{\phi w^{1-\sigma}(S_1/S_2)^{-(1-\sigma)}+1}\right)^{\frac{1-\alpha}{1-\sigma}}$$

The Jacobian matrix of system (55)-(57) evaluated at dispersion equilibrium (symmetric equilibrium) has the form

$$J^*_{1/2} = \begin{pmatrix} a & b & -b \\ -c & d & e \\ c & e & d \end{pmatrix} \tag{96}$$

with $a{=}\frac{1}{4}\left.\frac{\partial \Delta}{\partial L_1}\right|_{1/2}$, $b{=}\frac{1}{4S^*}\left.\frac{\partial \Delta}{\partial (S_1/S_2)}\right|_{1/2}$, $c{=}\epsilon S^*\left.\frac{\partial L_{H_1}}{\partial L_1}\right|_{1/2}$, $e{=}\epsilon\left.\frac{\partial L_{H_1}}{\partial (S_1/S_2)}\right|_{1/2}$ and $d{=}-\frac{gS^*}{CC}-e$. That is,

$$\begin{aligned}
a &= \tfrac{1}{2}\left(\tfrac{(1\text{-}\mu)\phi}{1+\phi}\text{-}\tfrac{\mu(1+\alpha(\sigma\text{-}1))\phi}{(\sigma\text{-}1)(1\text{-}\phi)}\right)\tfrac{\partial w}{\partial L_1}+\tfrac{\mu}{\sigma\text{-}1}\\
b &= \tfrac{1}{2}\left(\tfrac{(1\text{-}\mu)\phi}{1+\phi}\text{-}\tfrac{\mu(1+\alpha(\sigma\text{-}1))\phi}{(\sigma\text{-}1)(1\text{-}\phi)}\right)\tfrac{\partial w}{\partial S_1}+\tfrac{1}{4S^*}\left(\mu(1\text{-}\alpha)+(1\text{-}\mu)\tfrac{1\text{-}\phi}{1+\phi}\right)\\
c &= \epsilon S^*\left[\tfrac{\mu(1+\alpha(\sigma\text{-}1))\phi}{\sigma(1\text{-}\phi)^2}\left(\sigma\text{-}\tfrac{1\text{-}\phi}{2}\text{-}2(1\text{-}\alpha)(\sigma\text{-}1)\tfrac{\phi}{1+\phi}\right)\tfrac{\partial w}{\partial L_1}+1\text{-}\tfrac{\mu(1+\alpha(\sigma-1))(1+\phi)}{\sigma(1\text{-}\phi)}\right]\\
e &= \epsilon S^*\tfrac{\mu(1+\alpha(\sigma\text{-}1))\phi}{\sigma(1\text{-}\phi)^2}\left(\sigma\text{-}\tfrac{1\text{-}\phi}{2}\text{-}2(1\text{-}\alpha)(\sigma\text{-}1)\tfrac{\phi}{1+\phi}\right)\tfrac{\partial w}{\partial S_1}\text{-}\epsilon(1\text{-}\alpha)(\sigma\text{-}1)\tfrac{\mu(1+\alpha(\sigma\text{-}1))\phi}{\sigma(1\text{-}\phi^2)}\\
d &= -\tfrac{gS^*}{CC}-e<0
\end{aligned}$$

where $\partial w/\partial L_1$ and $\partial w/\partial S_1$ are given by (51) and (53).

The value $d+e=-gS^*/CC<0$ is an eigenvalue of matrix (96). Its characteristic polynomial is $P(\mathcal{L})=(d{+}e{-}\mathcal{L})\left[\mathcal{L}^2-(a+d-e)\mathcal{L}+2bc+a\,(d-e)\right]$. Then, the symmetric equilibrium is stable if and only if

$$2bc+a(d-e) > 0 \tag{97}$$

$$a+d-e < 0 \tag{98}$$