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Ranking scientists

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Abstract

Purpose: I provide a framework to construct rankings of scientists based on journal articles and their citations.
Design/Methodology/Approach: I assume a model in which the quantity and the impact of publications are economic goods and scientific committees derive utility from them. The committees's utility is therefore defined on the set of ordered pairs of the type (k, c_k) interpreted as "the kth most important publication of a given author has c_k cites." Within this framework, I derive the performance measure induced by utility maximization.
Findings: I prove that when quantity and impact are perfect substitutes, the induced performance index is the w-index in Woeginger (2008a) ["An axiomatic characterization of the Hirsch-index," *Mathematical Social Sciences,* **56**, 224–232.] In the case where quantity and impact are perfect complementaries, the induced performance index is the well known h-index in Hirsch (2005) ["An index to quantify an individual scientific research output," *Proceedings of the National Academy of Sciences,* **102** (46), 16569–16572.] Finally, when preferences are of Cobb-Douglas type, i.e. the trade off between quantity and impact equals the ratio between papers and cites, the induced index is Komulski's (2007) maxprod index ["MAXPROD–A new index for assesment of the scientific output of an individual, and a comparison with the h-index," *International Journal of Scientometrics, Informetrics, and Bibliometrics,,* **11** (1).]
Research limitations: The analysis of this model does not include some widely extended measures, as the criteria of average citations per paper.
Originality/Value: This model allows for a re-examination in terms of academic preferences of some scientific impact measures. Conversely, the model can help ranking designers to fit the needs of the institution using the ranking by first calibrating a utility function and then find the ranking induced by this utility.
JEL Classification: A11, C43, D70.

Keywords: Research productitvity, impact, performance index, utility maximization.

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"It is quality rather than quantity that matters..", Lucius Annaeus Seneca 5BC-65AD, Roman tragedian, philosopher, and counselor to Nero, Epistles.

1 Introduction

Although peer review is still the prevalent way by which research quality is judged, the idea that research assesment should be based on quantifiable data has become central in judging quality in science. As a result, a good deal of work has been concerned with quantifying at least the most obvious aspects of the impact and relevance of an individual's scientific research, namely publication and citation records, and to convert the details of a citation record into a scalar measure of quality. Thus, a scientist's full citation record is summarized by single-number criteria, such as the recently proposed Hirsch (2005) h-index. In its nearly five years of life, the h-index has been only the biggest splash in a flood of rating measures.[1] . Schools and labs are using such measures to help them make grants, bestow tenure, award bonuses or hire postdocs. The absence of an indisputable standard of measurament has triggered the study of axioms for performance measures in Woeginger (2008a,2008b), Quesada (2009a, 2009b) or Marchant (2009). This lack of consensus on how to measure simultaneously the diversity and impact of publications underlies the fact that different scientific committees may have different preferences over the magnitudes involved (papers and citations), which in turn lead to different indexes of scientific quality. Indeed, academic preferences can be context-dependent. Thus, if some academic department develops an incentive program addressed to tenured professors, it is likely to reward productivity (i.e. papers) more than impact (i.e. citations), because only productivity can be improved in the short term by working harder. However, if the same department wants to rank candidates for a opening position, probably it it will give more value to high impact papers than overall production.

My purpose is to examine the effect of performance preferences in the assessment of research output. I will make this point explicit by assuming a model in which the diversity and the impact of publications are economic goods and scientific committees derive utility from them. The committees's utility is therefore defined on the set of ordered pairs of the type (k, c_k) interpreted as "the kth most important publication of a given author has c_k cites." Within this framework, I derive the performance measure induced by this utility. The basic idea is that the overall scientific achievement of a scientist can be evaluated by the utility of the resulting papers.

[1] Panaretos and Malesios (2009) or Alonso *et al.* (2009) offer comprehensive reviews. Among economists, Ruan and Toll (2008) applicate this metric to rank economics departments in the Republic of Ireland.

2 Background and examples

In the bibliometrics literature, data for each scientist include the total number (n) of papers published over a given period T and the number of citations (c_i) for each paper $i \in \{1, \dots, n\}$. Papers are numbered in order of decreasing citations.[2] An individual research's career over T is therefore a vector $\mathbf{c} = (c_1, \dots, c_n)$ with non-negative integers $c_1 \geq \cdots \geq c_n$. Let $\mathcal{C}$ be the set of all such vectors. Given $\mathbf{c} \in \mathcal{C}$, an index to quantify this research assigns a chosen element $f(\mathbf{c}) \in \mathbb{R}$ to every possible output in $\mathcal{C}$ with the convention $f(\varnothing) = 0$.

For a given scientist with n published papers, some well known examples of quality indexes are:

- Total number of papers: $f_p(\mathbf{c}) = n$.

  The f_p score, or simply p-index, measures total production and disregards the information about impact or visibility of each paper.

- Total number of citations: $f_s(\mathbf{c}) = \sum_{i=1,\dots,n} c_i$.

  The f_s score or s-index measures total impact or visibility of an individual's research, and is independent of the production needed to obtain a given score.

- Average citations per paper: $f_{s/p} = f_s(\mathbf{c}) / f_p(\mathbf{c})$.

  Index $f_{s/p}$ rewards publishing widely cited papers and penalizes high productivity.

- Hirsch index: $f_h(\mathbf{c}) = \max\{i \mid c_i \geq i\}$. [Hirsch (2005)]

  According to index f_h, the publication of a "big hit" cannot make up for a scarce production. At the same time, the scientific value of a large amount of unimportant or rarely cited research cannot be improved by increasing total production, unless this additional production has a higher impact than the old one.

- Egghe index: $f_g(\mathbf{c}) = \min \left\{ n, \max\{i \mid \sum_{j=1}^{i} c_j \geq i^2\} \right\}$.[Egghe (2006a, 2006b)]

  Like the Hirsch index, the f_g-index is insensitive to low-impact publications. However, the f_g-index is not insensitive to the level of the excepcional papers, and, thus the skewer the citation distribution, the higher its score will be.

In the next section, I present a formal analysis that attempts to capture some of the issues raised here.

[2] Publications with the same number of citations are given different ranking (the exact number does not matter).

3 From preferences to rankings

Quoting Ball (2005):

"The election procedures of scientific academies are often seen as opaque, clubby and capricious".

A way to silence those complaints is inventing a measure of research perfomance or index. The examples above show some which have received most attention, whereas many others have had little response. In any case, the merits of each measure rely largely on intuitive arguments and value judgements. In fact, several are often only vaguely related to the intuitive ideas they purport to index, and many are so complex that is is difficult to discover what, if anything, they are measuring.[3]

A different approach the this problem, which I shall develop here, elaborates on the assumption that scientific knowledge is an economic commodity and scientific papers are the basic medium for its dissemination and exchange. As consumers of this commodity, academies have preferences over papers assumed to be exogenous. Thus, I treat the preferences of scientific academies over papers as the primitive and proceed by deriving choices among scientists from this relation through preference maximization.

To formalize this approach, I shall re-write the publication record of a scientist $\mathbf{c} = (c_1, \ldots, c_n)$ as the set $S_{\mathbf{c}} = \{(k, c_k)\}_{k=1,\ldots,n}$ in $\mathbb{R}^2_+$. Each point in $S_{\mathbf{c}}$ represents a paper characterized by two attributes. The first component represents the rank of the paper within the scientist's total production, whereas the second component denotes its citation score.[4] The objectives of the academia are summarized in a preference-or-indifference relation $\succsim$ on $K \times X$ where K and X are either the set $\mathbb{N}$ or an interval of positive numbers. The preferences $\succsim$ are assumed to be complete, transitive and continuous in the relative usual topologies when K and X are an interval, or the discrete topology when they are $\mathbb{N}$. Any continuous utility function $u : \mathbb{R}^2_+ \longrightarrow \mathbb{R}$ that represents $\succsim$ reflects the judgement values that the academia applies to the novelty and diversity of a paper within a scientist career, embodied in the value of k, and to the global influence or impact of this paper, given by c_k. Thus, for any given $\mathbf{c}$, the value of her kth most cited publication is $u(k, c_k)$. Implicit in this formulation is an anomymity assumption: Academia cares about the rank and the impact of a paper—not who is the author, hence the value function is author-independent. Since it is better to publish more than less and that the citation count of a paper is a positive measure of its impact, we shall assume u, nondecreasing in both arguments. Namely:

[3] In spite of this menu of alternative indices, all empirical studies that have tested them have reported high correlations coefficients. Apparently, this indicates a redundancy among the various measures of achievement.

[4] Notice that this formulation allows easily for both the rank and the number of citations to be not natural numbers, for instance when one uses a correction factor to account for the length of the paper or citations are counted fractionally in multi-authored papers.

(A1) **"diversity is desirable"** If $i > j$ then $u(i, c_i) \geq u(j, c_j)$;

(A2) **"Impact is valuable"** If $c_i' > c_i$ then $u(i, c_i') \geq u(i, c_i)$.

In consumer theory, it often seems reasonable to add a convexity condition. Convex preferences will lie between the following two polar cases:

(i) Perfect substitutes Academic preferences can be represented by a utility function of the linear family $u(i, c_i) = a_1 i + a_2 c_i$ with $a_1,\ a_2 \geq 0$, i.e. there is always a constant rate of substitution $-a_1/a_2$ between the diversity and the apparent impact of scientific output.

(ii) Perfect complementaries Academic preferences can be represented by a utility function of the Leontief family $u(i, c_i) = \min\left\{\frac{i}{a_1}, \frac{c_i}{a_2}\right\}$, with $a_1,\ a_2 > 0$, i.e. there is no trade off between diversity and impact, both goods are consumed together.

A sort of "balanced" preferences between these extremes examples is:

(iii) Cobb-Douglas Academic preferences can be represented by a product utility function $u(i, c_i) = ic_i$. In this case the rate of substitution between productivity and impact is simply i/a_i.

However, I know of no persuasive argument for convexity within this context and therefore I let this issue open to exception. Now, given u, utility maximization defines the induced index f_u of a scientist:

Induced utility index: For $\mathbf{c} \in \mathcal{C}$, $f_u(\mathbf{c}) = \max\{u(z) \mid z \in S_{\mathbf{c}}\}$

Notice that since u is determined up to a monotonically increasing transformation, the ranking among scientists is uniquely characterized by the preference relation $\succsim$. There are, however, infinite indices which induce the same ranking, all of them induced by an increasing transformation of u.

4 A graphical depiction

It is helpful to be able to visualize the preferences-based model at work in the space of commodities $\mathbb{R}_+ \times \mathbb{R}_+$. Geometrically, given academic preferences $\succsim$ and fixed a utility representation u, the induced index of scientist $\mathbf{c}$ is given by the value of the highest indifference curve intersecting $S_{\mathbf{c}}$. Thus, $f_u(\mathbf{c}) = u(k^*, c_{k^*})$ for any paper (k^*, c_{k^*}) in this intersection. Now, given $\mathbf{c} = (c_1, \ldots, c_n) \in \mathcal{C}$, I shall define $x_{\mathbf{c}} : \mathbb{R}_+ \longrightarrow \mathbb{R}$ as $x_{\mathbf{c}}(\mu) = c_i$, where $i = \lceil \mu \rceil = \min\{j \mid j \geq \mu\}$.provided that $c_i = 0$ if $i > n$. Notice that the area under $x_{\mathbf{c}}$, $\int_0^\infty x_{\mathbf{c}}$, yields the total number of citations or index $f_s(\mathbf{c})$, which is independent of the academic preferences as long as they are monotonic. Many other indices are utility-induced. If preferences are convex, any induced index coincides with the highest indifference curve that intersects the graph of $x_{\mathbf{c}}$ at a point for which the number of cites

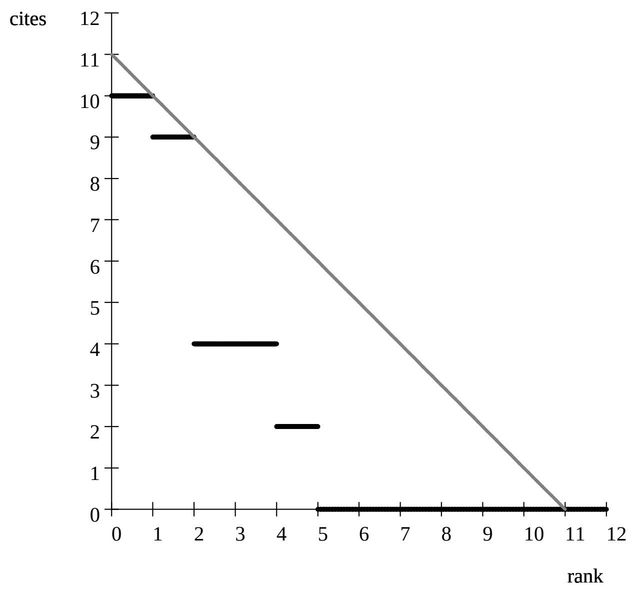

Figure 1: Figure 1A The induced utility index when $u(i, c_i) = i + c_i$.

is strictly positive. In the case where academic preferences are perfect substitutes (case 1 in the previous section), indifference curves are straight lines with slope $-a_1/a_2$. If $a_1 = a_2 = 1$, the highest indifference curve intersecting the graph of $x_{\mathbf{c}}$ draws an isosceles right angled triangle with the axis. The legs of this triangle yield the induced index $f_u(\mathbf{c})$. In this symmetric case the induced index coincides with the w-index defined in Woeginger (2008a) for integer values (see Conclusion 1 below for a formal statement of this result). As an example, figure 1A below plots the function $x_{\mathbf{c}}$ in the case in which the scientist $\mathbf{c}$ has one published paper with 10 cites, one with 7 cites, two with 4 cites and one with 2 cites, i.e. $\mathbf{c} = (10, 9, 4, 4, 2)$ and $S_{\mathbf{c}} = \{(1, 10), (2, 9), (3, 4), (4, 4), (5, 2)\}$. The induced utility index or w-index is $f_u(\mathbf{c}) = u(1, 10) = u(2, 9) = 11$.

In the case where diversity and impact are perfect complementaries and $a_1 = a_2 = 1$, (see case 2 above) the induced utility index is given by the utility of the point (k^*, k^*) in which the diagonal line $c_i = i$ intersects the graph of $x_{\mathbf{c}}$. If we draw horizontal and vertical lines from this point until reaching the axis, the side length of the resulting square yields the well known h-index in Hirsch (2005). Now, scientist $\mathbf{c} = (10, 9, 4, 4, 2)$ has induced utility or Hirsch index $f_u(\mathbf{c}) = u(4, 4) = 4$.

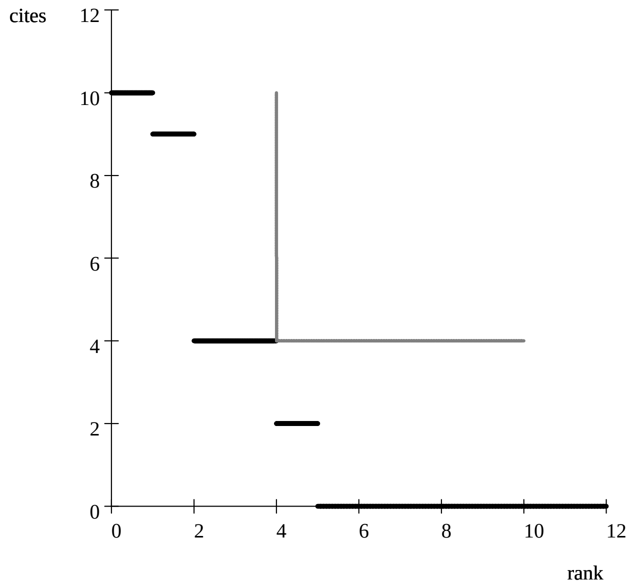

Figure 1B The induced utility index when $u(i, c_i) = \min\{i, c_i\}$.

Notice that by allowing $a_1 \neq a_2$, in the continuous case the induced index is characterized by the utility of the point which lies in the intersection of the line $c_i = a_1 i/a_2$ and the boundary of the area below $S_\mathbf{c}$.[5] Hence, by setting $\alpha = a_1/a_2$ and $a_1 = 1$, the induced index f_u coincides with the h_α generalization of the Hirsch index proposed in van Eck and Waltman (2008) (Conclusion 2 below).

The Cobb-Douglas case above also has a simple geometry: $f_u(\mathbf{c})$ is given by the utility $k^* c_{k^*}$ where (k^*, c_{k^*}) is a point in $S_\mathbf{c}$ through which we can draw a line with the property that its midpoint in the orthant is precisely (k^*, c_{k^*}). Notice that since $f_u(\mathbf{c})$ coincides, by definition, with the highest value among values ic_i. In our example, this yields $f_u(\mathbf{c}) = u(2, 9) = 18$ (see figure 1C). This induced index coincides with the MAXPROD index introduced by Kosmulski (2007) (Conclusion 3 below).

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[5] Notice that the intersection of $c_i = a_1 i/a_2$ and the graph of $x_\mathbf{c}$ is not guaranteed since $x_\mathbf{c}$ is not continuous.

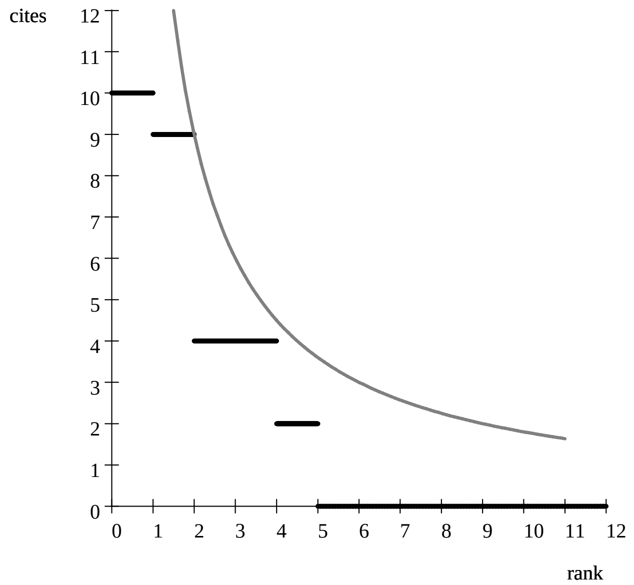

Figure 1C The induced utility index when $u\,(i, c_i) = ic_i$.

5 Conclusions and suggestions for further research

The following are applications of this approach to the specific families of utility functions presented in the previous section.

Conclusion 1: In the case where the academic preferences can be represented by the utility $u\,(i, c_i) = a_1 i + a_2 c_i$, the index f_u generalizes the w-index in Woeginger (2008a).

Proof. The w-index assigns to $\mathbf{c} = (c_1, \ldots, c_n)$ the value $w(\mathbf{c}) = \max\{k \mid i + c_i \geq k + 1, \forall i \leq k\}$. Therefore, it coincides with the value of the largest indifference curve of $u\,(i, c_i) = i + c_i$ which contains a point in $S_{\mathbf{c}}$, and hence, $w(\mathbf{c}) = f_u(\mathbf{c})$ for $u\,(i, c_i) = i + c_i$.

Conclusion 2: In the case where the academic preferences can be represented by the utility $u\,(i, c_i) = \min\{i/a_1, c_i/a_2\}$, the index f_u is the h_α generalization of the Hirsch index in van Eck and Waltman (2008).

Proof. For any given $\alpha \in (0, \infty)$, the h_α-index is defined as $h_\alpha(\mathbf{c}) = \max\{\mu \mid x(\mu) \geq \alpha\mu\}$. where $x(\mu) = c_i$.for $i = \min\{j \mid j \geq \mu\}$. Let μ^* be the solution of $x(\mu) = \alpha\mu$. By construction, $h_\alpha(\mathbf{c}) = \mu^*$. It readily follows that by setting $a_1/a_2 = \alpha$ there is i such that $\min\{i/a_1, c_i/a_2\} = \mu^*$ and $\min\{j/a_1, c_k/a_2\} > \mu^*$ for all $j > i$. Thus, $h_{a_1/a_2}(\mathbf{c}) = f_u(\mathbf{c})$ for $u\,(i, c_i) = \min\{i/a_1, c_i/a_2\}$ as stated.

Conclusion 3: In the case where the academic preferences can be represented by the utility $u(i, c_i) = ic_i$, the index f_u is Kosmulski's (2007) MAXPROD.

Proof. Omitted.

The analysis of the discussed model may be extended and modified in numerous ways. For instance, it should be possible a re-examination of the set of axioms used in axiomatizations of some scientific impact measures.[6] Conversely, the model can help ranking designers to fit the needs of the institution using the ranking. That is, according to the aim of the institution using a ranking of scientists, one may first calibrate a utility function and then find the ranking induced by this utility. In this context, I believe Kosmulski's (2007) MAXPROD index to be extremely interesting. As I said before, within the class of convex preferences, preferences leading to the h- and the w-indices represent extreme cases in which the trade-off between diversity and impact is either non-existent or constant. Indeed, one of the main criticisms about the h-index is that while it de-emphasizes singular succesful publications in favour of sustained productivity, it may do so too strongly. By assuming preferences with a non-trivial rate of substitution, as Kosmulski's index does, one would expect the index to be more "balanced."

Of course, there are indices that lie outside of this approach, for instance the well known average citations per paper index. In fact any induced utility index must satisfy a sort of independence of "irrelevant" publications and/or citations. To see this, notice that if for scientists $\mathbf{c}$ and $\mathbf{c}'$ we have $S_{\mathbf{c}'} \subset S_{\mathbf{c}}$ and $f_u(\mathbf{c}) \in S_{\mathbf{c}'}$, by utility maximization, it must follow that $f_u(\mathbf{c}') = f_u(\mathbf{c})$ for any given u. If we assume this property as an axiom, it would be interesting to check under which conditions any index satisfying this requirement is compatible with the existence of underlying preferences over papers. I hope to deal with this question in another paper.

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[6] For instance, in Woeginger (2008b the h- and the w-indices are characterized by imposing a symmetry property which, in the author's words:

> *"...esentially imposes that the number of citations and the number of publications should be treated in the same way and should be measured in the same scale".*

In the induced utility model, Woeginger's assumption amounts to require from academic preferences a symmetric utility representation.

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